H_∞ Control of Dead-Time Systems Based on a Transformation ⋆

Qing-Chang Zhong [a]

[a] *Faculty of Mechanical Engineering, Technion-Israel Institute of Technology, Technion City, Haifa, 32000, Israel*

Abstract

This paper presents a transformation that makes all the robust control problems of dead-time systems able to be solved similarly as in the finite dimensional situations. With trade-off of the performance, some advantages obtained are: (i) The controller has a quite simple and transparent structure; (ii) There are no any *additional* hidden modes in the Smith predictor and, hence, there is no *additional* hidden possibility to destabilize the system; (iii) it can be applied to systems with long dead-time without any difficulty. Hence, the practical significance is obvious.

Key words: Dead-time systems, dead-time compensator, H_∞ control, Smith predictor, performance index

1 Introduction

Dead-time systems are systems in which the action of control inputs takes a certain time before it affects the measured outputs. The typical dead-time systems are those with input delays. It is well known that it is very difficult to control such systems. Smith predictor (Smith, 1957) is the first effective way to control such systems and many modified Smith predictors were presented (Palmor, 1996). In recent years, many researchers are interested in the optimal

⋆ This research was supported by THE ISRAEL SCIENCE FOUNDATION (Grant No. 384/99) founded by The Israel Academy of Sciences and Humanities.
Email address: zhongqc@ic.ac.uk (Qing-Chang Zhong).

[1] Qing-Chang Zhong is currently at Imperial College, Exhibition Road, London SW7 2BT, UK. *Tel*: 44-20-759 46295, *Fax*: 44-20-759 46282, *Email*: zhongqc@ic.ac.uk, *URL*: http://come.to/zhongqc.

Preprint submitted to Elsevier Preprint

control of dead-time systems, especially H_∞ control, i.e., to find a controller to internally stabilize the system (if so, called an admissible controller) and to minimize the H_∞-norm of the transfer matrix from the external input signals (such as noises, disturbances and reference signals) to the output signals (such as controlled signals and tracking errors).

With the help of the modified Smith predictor, many robust control problems for dead-time systems, such as robust stability, tracking and model-matching and input sensitivity minimization, can be solved as in the finite dimensional situations (Meinsma and Zwart, 2000; Nobuyama, 1992). However, the sensitivity minimization, the mixed sensitivity minimization and/or the standard H_∞ control problems cannot be solved in this way. Very recently, notable results were presented in (Mirkin, 2000; Meinsma and Zwart, 2000; Tadmor, 2000; Nagpal and Ravi, 1997; Zhong and Mirkin, 2001; Zhong, 2002) using different methods. The results in (Mirkin, 2000; Meinsma and Zwart, 2000; Zhong and Mirkin, 2001), which are formulated in the form of a modified Smith predictor, are quite elegant and the ideas are very tricky, but the controllers are too involved. Specifically, the Smith predictor is quite complex and, even more, relates to the performance level γ. Hence, there exist some problems to apply the method to systems with long dead-time, as pointed out by the authors in (Meinsma and Zwart, 2000). Another disadvantage is that the predictor (see, F_{stab} in (Meinsma and Zwart, 2000, Theorem 5.3), $\Delta_{\alpha,\infty}$ in (Mirkin, 2000, Theorem 2) or $\Delta(s)$ in (Zhong and Mirkin, 2001, Theorem 2)) always includes *additional* unstable hidden modes, even with stable plants, because the hidden modes are the eigenvalues of a Hamiltonian matrix. It has been pointed out in (Van Assche *et al.*, 1999; Manitius and Olbrot, 1979) that such hidden modes are not safe and tend to destabilize the system when implemented. Hence, the practical significance of these results may be limited.

This paper presents a transformation on the closed-loop transfer matrix of dead-time systems. In fact, it is a new H_∞ performance evaluation scheme for dead-time systems. With this transformation, all robust control problems can be solved as in the finite dimensional situations. The controller obtained has a quite simple and transparent structure with a modified Smith predictor. The resulted Smith predictor only depends on the real plant and is independent of the performance level γ and of the performance evaluation scheme. There do not exist any *additional* hidden modes and, hence, there is no *additional* hidden possibility to destabilize the system. This method can be applied to systems with long dead-time without difficulty. The cost to pay for these advantages is some performance losses from the conventional H_∞ control point of view. When the dead-time h becomes 0, the transformation becomes null. Hence, it can be regarded as an extension of the conventional performance evaluation scheme for dead-time systems. This transformation does not affect the original system and is quite ease to be applied because it just, *virtually*, subtracts an finite impulse response (FIR) block from the original system.

The rest of the paper is organized as follows: the transformation (or the new performance evaluation scheme) is presented in Section 2; the solution to H_∞ control of dead-time systems with an input/output delay is given in Section 3; an example is given in Section 4.

Notation Assume

$$G(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

is a rational transfer matrix $G(s) = D + C(sI - A)^{-1}B$. Truncation operator $\tau_h\{G\}$ and completion operator $\pi_h\{e^{-sh}G\}$ are defined, respectively, as

$$\tau_h\{G\} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] - e^{-sh}\left[\begin{array}{c|c} A & e^{Ah}B \\ \hline C & 0 \end{array}\right] \doteq G(s) - e^{-sh}\tilde{G}(s),$$

$$\pi_h\{e^{-sh}G\} = \left[\begin{array}{c|c} A & B \\ \hline Ce^{-Ah} & 0 \end{array}\right] - e^{-sh}\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \doteq \hat{G}(s) - e^{-sh}G(s).$$

They are slightly different with those defined in (Mirkin, 2000). Note that these two operators map any rational transfer matrix G into an FIR block. The impulse response of $\tau_h\{G\}$ is the truncation of the impulse response of G to $[0, h]$. The impulse response of $\pi_h\{e^{-sh}G\}$, which is also supported on $[0, h]$, is the only continuous function on $[0, h]$ with the following property: if we add it to the impulse response of $e^{-sh}G(s)$, which is supported on $[h, \infty)$, we obtain the impulse response of a rational transfer matrix, denoted above by $\hat{G}$.

2 The Transformation

The general control setup for dead-time systems with a input or output delay is shown in Figure 1, where

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}.$$

The closed-loop transfer matrix from w to z can be formulated as

$$T_{zw}(s) = P_{11} + e^{-sh}P_{12}K(I - e^{-sh}P_{22}K)^{-1}P_{21}.$$

This means there exists an instant response without delay through the path P_{11}. A clearer equivalent structure is shown in Figure 2. It's not difficult to recognize that, during the period $t = 0 \sim h$ after w is applied, the output z is *not* controllable (i.e., not changeable by the control action) and is *only* determined by P_{11} (and, of course, w). However, the response during this period may dominate the system performance index. This means the performance index is likely dominated by the response we cannot control. This is what we do not want. It does not make sense to include such an uncontrollable part in the performance index. Hence, we should eliminate the response during this period when evaluating the system performance. This is a key idea in this paper. In general, it is impossible to eliminate the instant response by simply introducing a suitable P_{11}. A possible way to implement this idea is proposed below. There may be other ways to implement this idea.

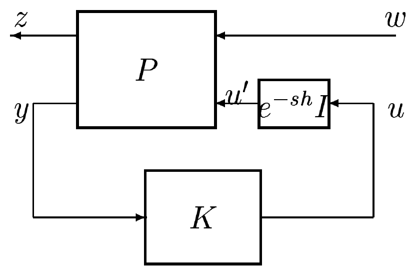


Figure 1. General control setup of dead-time system

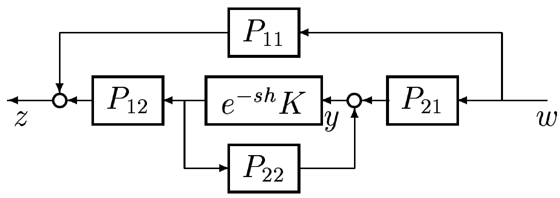


Figure 2. Equivalent structure

Define an FIR block

$$\Delta_1(s) = \tau_h\{P_{11}\} = P_{11}(s) - \tilde{P}_{11}(s)e^{-sh},$$

which is exactly the uncontrollable part in $T_{zw}(s)$. Subtract it from the feedforward path P_{11}, as shown in Figure 3. We have

$$T_{zw}(s) = \Delta_1(s) + T_{z'w}(s), \quad (1)$$

where

$$T_{z'w}(s) = e^{-sh}\{\tilde{P}_{11} + P_{12}K(I - e^{-sh}P_{22}K)^{-1}P_{21}\}.$$

The fact (1) was recognized in (Mirkin and Raskin, 1999) to parameterize all stabilizing dead-time controllers. There is no instant response in $T_{z'w}(s)$. The only difference between $T_{z'w}(s)$ and $T_{zw}(s)$ is the FIR block $\Delta_1(s)$, which is not a part of the real control system but an artificial part resulted from the performance evaluation scheme. It is this FIR block that makes the control problems so complex and difficult to be solved. It has been shown in (Mirkin, 2000) that the achievable minimal performance index $\|T_{zw}\|_\infty$ is larger than $\|\Delta_1\|_\infty$. Hence,

$$\|\Delta_1\|_\infty < \|T_{zw}\|_\infty \leq \|\Delta_1\|_\infty + \|T_{z'w}\|_\infty . \tag{2}$$

We optimizes $\|T_{z'w}\|_\infty$ but not $\|T_{zw}\|_\infty$ in this paper. Once $\|T_{z'w}\|_\infty$ is minimized, the achievable performance $\|T_{zw}\|_\infty$ is not larger than $\|\Delta_1\|_\infty + \|T_{z'w}\|_\infty$ and so is the optimal performance. Using the inequality (2), one can estimate how far the proposed sub-optimal controller is away from the optimal controller.

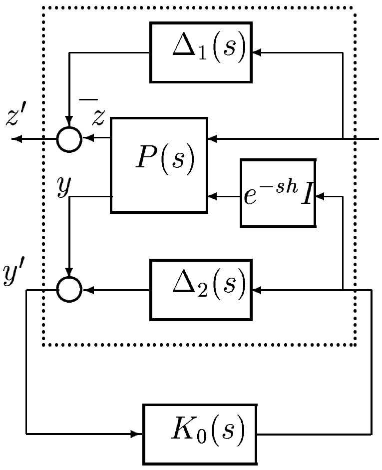


Figure 3. Graphic interpretation of the transformation

3 H_∞ Control of Systems with a Single Delay

Assume that the realization of the rational part of the generalized process in Figure 1 is taken to be of the form

$$P(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$$

and the following standard assumptions hold:

(A1) (A, B_2) is stabilizable and (C_2, A) is detectable;

(A2) $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank $\forall \omega \in R$;

(A3) $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank $\forall \omega \in R$;

(A4) $D_{12}^* D_{12} = I$ and $D_{21} D_{21}^* = I$.

Assumption (A4) is made just to simplify the exposition. In fact, only the non-singularity of the matrices $D_{12}^* D_{12}$ and $D_{21} D_{21}^*$ is required.

Consider a Smith predictor-type controller

$$K(s) = K_0(s)(I - \Delta_2(s)K_0(s))^{-1},$$

as shown in Figure 3, in which the predictor is designed as

$$\Delta_2(s) = \pi_h\{e^{-sh}P_{22}\} = \left[\begin{array}{c|c} A & B_2 \\ \hline C_2 e^{-Ah} & 0 \end{array}\right] - e^{-sh}\left[\begin{array}{c|c} A & B_2 \\ \hline C_2 & D_{22} \end{array}\right],$$

then the system can be re-formulated as

$$\begin{bmatrix} z'' \\ y' \end{bmatrix} = \widetilde{P}(s) \begin{bmatrix} w \\ u \end{bmatrix}$$
$$u = K_0(s) y'$$

with $e^{-sh} z'' \doteq z'$, where

$$\widetilde{P}(s) = \left[\begin{array}{c|cc} A & e^{Ah}B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ C_2 e^{-Ah} & D_{21} & 0 \end{array}\right]$$

is free of dead-time (but delay-dependent). The closed-loop transfer function from w to z' is

$$T_{z'w}(s) = e^{-sh} T_{z''w}(s) = e^{-sh} \mathcal{F}_l(\widetilde{P}(s), K_0(s)). \tag{3}$$

Hence, the H_∞ control problem

$$\|T_{z'w}(s)\|_\infty < \gamma$$

is converted to

$$\left\|\mathcal{F}_l(\tilde{P}(s), K_0(s))\right\|_\infty < \gamma.$$

This is a finite dimensional problem which can be solved with known results. Since this is for the general setup of systems with an input/output delay, all the H_∞ control problems (such as robust stability, tracking and model-matching, input sensitivity minimization, output sensitivity minimization, mixed sensitivity minimization and the standard H_∞ control problem etc.) can be solved similarly as in the finite dimensional situations.

The solution, which is given in Theorem 1 below, involves two Hamiltonian matrices:

$$H_h = \begin{bmatrix} A & \gamma^{-2} e^{Ah} B_1 B_1^* e^{A^* h} \\ -C_1^* C_1 & -A^* \end{bmatrix} - \begin{bmatrix} B_2 \\ -C_1^* D_{12} \end{bmatrix} \begin{bmatrix} D_{12}^* C_1 & B_2^* \end{bmatrix},$$

$$J_h = \begin{bmatrix} A^* & \gamma^{-2} C_1^* C_1 \\ -e^{Ah} B_1 B_1^* e^{A^* h} & -A \end{bmatrix} - \begin{bmatrix} e^{-A^* h} C_2^* \\ -e^{Ah} B_1 D_{21}^* \end{bmatrix} \begin{bmatrix} D_{21} B_1^* e^{A^* h} & C_2 e^{-Ah} \end{bmatrix}.$$

Theorem 1 *There exists an admissible main controller such that* $\|T_{z'w}(s)\|_\infty < \gamma$ *in Figure 3 iff the following three conditions hold:*

(i) $H_h \in dom(Ric)$ *and* $X = Ric(H_h) \geq 0$;

(ii) $J_h \in dom(Ric)$ *and* $Y = Ric(J_h) \geq 0$;

(iii) $\rho(XY) < \gamma^2$.

Moreover, when the conditions hold, one such main controller is

$$K_0(s) = \left[\begin{array}{c|c} A_h & -L_h \\ \hline F_h Z_h & 0 \end{array}\right],$$

where

$$A_h = A + L_h C_2 e^{-Ah} + \gamma^{-2} Y C_1^* C_1 + \left(B_2 + \gamma^{-2} Y C_1^* D_{12}\right) F_h Z_h,$$

$$F_h = -(B_2^* X + D_{12}^* C_1), \;\; L_h = -(Ye^{-A^*h} C_2^* + e^{Ah} B_1 D_{21}^*), \;\; Z_h = (I - \gamma^{-2} YX)^{-1}.$$

Furthermore, the set of all admissible main controllers such that $\|T_{z'w}(s)\|_\infty < \gamma$ *can be parameterized as*

$$K_0(s) = \mathcal{F}_l(M(s), Q(s)),$$

where

$$M(s) = \left[\begin{array}{c|cc} A_h & -L_h & B_2 + \gamma^{-2} Y C_1^* D_{12} \\ \hline F_h Z_h & 0 & I \\ -\left(C_2 e^{-Ah} + \gamma^{-2} D_{21} B_1^* e^{A^*h} X\right) Z_h & I & 0 \end{array}\right]$$

and $Q(s) \in H_\infty$, $\|Q(s)\|_\infty < \gamma$.

PROOF. First of all, check if $\tilde{P}(s)$ meets the standard assumptions (A1-A4).

(A1) $(C_2 e^{-Ah}, A)$ is detectable because $A + e^{Ah} L C_2 e^{-Ah} \sim A + LC_2$ and (C_2, A) is detectable;

(A3) $\begin{bmatrix} A - j\omega I & e^{Ah} B_1 \\ C_2 e^{-Ah} & D_{21} \end{bmatrix}$ has full row rank $\forall \omega \in R$ because $\begin{bmatrix} A - j\omega I & e^{Ah} B_1 \\ C_2 e^{-Ah} & D_{21} \end{bmatrix}$ $\sim \begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank $\forall \omega \in R$.

The assumptions (A2) and (A4) remain unchanged. Hence, $\tilde{P}(s)$ meets all the standard assumptions. Theorem 5.1 in (Green *et al.*, 1990) can be directly used to solve this problem. Substitute $\tilde{P}(s)$ into that theorem, then the above result can be obtained with ease.

Remark 2 *It is worth noting that, under this transformation, either* $D_{11} \neq 0$ *or* $D_{22} \neq 0$ *does not make the problem more complex. In fact, when* $h = 0$, $\Delta_2(s) = -D_{22}$ *is the common controller transformation to make* $D_{22} = 0$ *(Zhou, 1998).*

4 Example

Consider the example studied in (Meinsma and Zwart, 2000), see Figure 4, where

$$P_r(s) = \frac{1}{s-1} \text{ and } P_{rd}(s) = \frac{s-1}{s+1},$$

$$W_1(s) = \frac{2(s+1)}{10s+1} \text{ and } W_2(s) = \frac{0.2(s+1.1)}{s+1}.$$

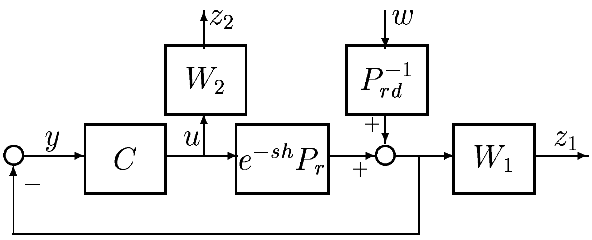


Figure 4. Setup for mixed sensitivity minimization

This problem can be arranged as a standard problem with

$$P(s) = \left[\begin{array}{c|c} \frac{2(s+1)^2}{(10s+1)(s-1)} & \frac{2(s+1)}{(10s+1)(s-1)} \\ 0 & \frac{0.2(s+1.1)}{s+1} \\ \hline -\frac{s+1}{s-1} & -\frac{1}{s-1} \end{array}\right].$$

Here, $P_{22} = -\frac{1}{s-1}$, $P_{11} = \begin{bmatrix} \frac{2(s+1)^2}{(10s+1)(s-1)} \\ 0 \end{bmatrix}$. $W_2(s)$ is delayed as $W_2 e^{-sh}$. This does not affect the result.

The predictor is designed as

$$\Delta_2(s) = -\frac{e^{-h} - e^{-sh}}{s-1}.$$

There is no additional hidden mode; the only hidden mode is the same as the mode of the real plant P_{22}. However, the predictor obtained in (Meinsma and Zwart, 2000) is

$$F_{stab}(s) = -\frac{10.75s^2 + 0.245s - 0.3298}{(s-1)(12.03s^2+1)} + \frac{13.16s^2 - 0.1316}{(s-1)(12.03s^2+1)} e^{-sh}.$$

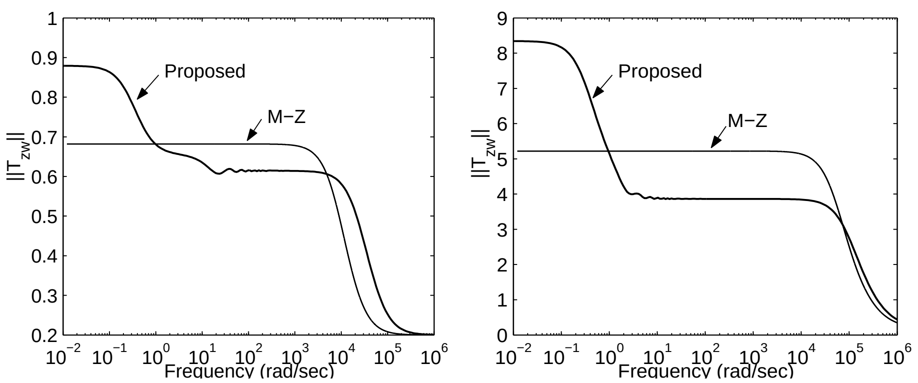
Table 1
Performance Comparison

h	M-Z's $\|T_{zw}(s)\|_\infty$	Proposed $\|T_{z'w}(s)\|_\infty$	Proposed $\|T_{zw}(s)\|_\infty$
0.2	*0.68*	0.58	*0.88*
2	*5.22*	3.86	*8.34*
5	109.5*	78.63	*183.7*

*Note: For long dead-time h, e.g. $h > 3.2$sec, some matrices are close to singular or badly scaled; the result of M-Z is likely inaccurate.

It is much more complex. There are three hidden modes: one is the same as the mode of the real plant P_{22} and the other two $s_{1,2} = \pm 0.2883j$ are additional.

The achievable performance is listed in Table 1 for different delays h. The performance of the proposed method degrades not too much to pay for the advantages obtained. The frequency responses of both cases are shown in Figure 5. Although the H_∞-norm achieved in this paper is a little bit larger than that achieved by the method of Meinsma and Zwart (noted as M-Z in figures), the proposed method obtains better performance in a quite broad frequency band ranging from $1 rad/sec$ to about $10,000 rad/sec$. From the engineering point of view, this solution is much better. The singular value plot of the uncontrollable part $\Delta_1(s)$ is shown in Figure 6. It is clear that the high gain of T_{zw} at low frequency is contributed (or dominated) by the large singular value of $\Delta_1(s)$ at low frequency.

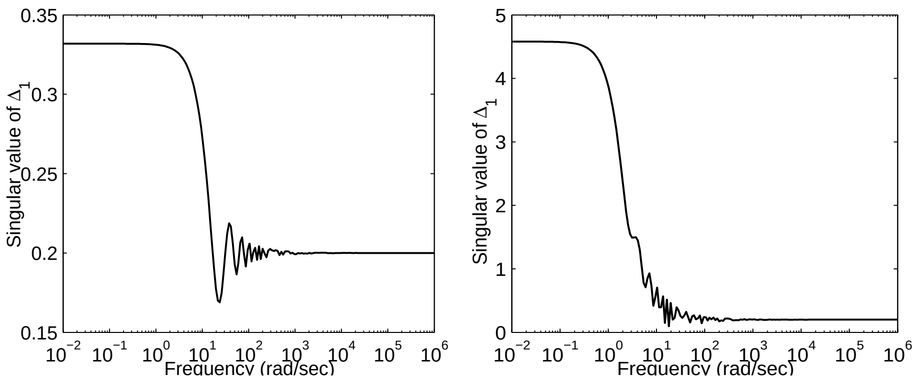


(a) $h = 0.2s$ (b) $h = 2s$

Figure 5. Comparison of $\|T_{zw}(j\omega)\|$

5 Conclusions

This paper introduces a transformation (in fact, a new performance evaluation scheme) for the H_∞ control of dead-time systems. It considerably simplifies the H_∞ control problem of dead-time systems and has many advantages. In



(a) $h = 0.2s$ (b) $h = 2s$

Figure 6. The singular value plot of $\Delta_1(s)$

particular, the practical significance is obvious. Since this is not a solution to the original H_∞ control problem of time delay systems there may exist some performance losses from the conventional H_∞ control or mathematical point of view. An inequality has been given to estimate how far the sub-optimal controller is away from the optimal controller; more accurate analysis of the performance loss is undertaken.

Acknowledgment

The author would like to thank Dr. Leonid Mirkin, Dr. George Weiss, Prof. Kemin Zhou and the anonymous reviewers for their helpful suggestions.

References

Green, M., K. Glover, D. Limebeer and J. Doyle (1990). A J-spectral factorization approach to H_∞ control. *SIAM J. Control Optim.* **28**(6), 1350–1371.

Manitius, A.Z. and A. W. Olbrot (1979). Finite spectrum assignment problem for systems with delays. *IEEE Trans. Automat. Control* **24**(4), 541–553.

Meinsma, G. and H. Zwart (2000). On $\mathcal{H}_\infty$ control for dead-time systems. *IEEE Trans. Automat. Control* **45**(2), 272–285.

Mirkin, L. (2000). On the extraction of dead-time controllers from delay-free parametrizations. In: *Proc. of 2nd IFAC Workshop on Linear Time Delay Systems.* Ancona, Italy. pp. 157–162.

Mirkin, L. and N. Raskin (1999). State-space parametrization of all stabilizing dead-time controllers. In: *Proc. of the 38th Conf. on Decision & Control.* Phoenix, Arizona, USA. pp. 221–226.

Nagpal, K. M. and R. Ravi (1997). H_∞ control and estimation problems with delayed measurements: State-space solutions. *SIAM J. Control Optim.* **35**(4), 1217–1243.

Nobuyama, E. (1992). Robust stabilization of time-delay systems via reduction to delay-free model matching problems. In: *Proc. of the 31th IEEE Conference on Decision & control*. Vol. 1. Tucson, Arizona. pp. 357–358.

Palmor, Z. J. (1996). Time-delay compensation — Smith predictor and its modifications. In: *The Control Handbook* (S. Levine, Ed.). pp. 224–237. CRC Press.

Smith, O. J. M. (1957). Closer control of loops with dead time. *Chem. Eng. Progress* **53**(5), 217–219.

Tadmor, G. (2000). The standard H_∞ problem in systems with a single input delay. *IEEE Trans. Automat. Control* **45**(3), 382 –397.

Van Assche, V., M. Dambrine and J.F. Lafay (1999). Some problems arising in the implementation of distributed-delay control laws. In: *Proc. of the 38th IEEE Conference on Decision & control*. Phoenix, Arizona, USA. pp. 4668–4672.

Zhong, Q.C. (2002). Frequency domain solution to the delay-type Nehari problem. In: *Proc. of the 15th IFAC World Congress*. Barcelona, Spain. An extended version,
which is available at http://www.ee.ic.ac.uk/CAP/Reports/reports.html, was submitted to Automatica.

Zhong, Q.C. and L. Mirkin (2001). On the standard H_∞ control of processes with a single delay. Submitted to *IEEE Trans. Automat. Control*, available at http://www.ee.ic.ac.uk/CAP/Reports/reports.html.

Zhou, K. (1998). *Essentials of Robust Control*. Prentice-Hall. Upper Saddle River, N.J.