Control of Integral Processes with Dead Time.

Part 3: Deadbeat Disturbance Response*

Qing-Chang Zhong†

Abstract

A deadbeat disturbance response of integral processes with dead-time is obtained by intentionally using two adjustable delay elements in the controller. These delays are tuning parameters of the controller. The shorter delay is optimally determined to minimize the robustness indicator (equivalently, to maximize the allowable uncertainty bound) while the longer delay (*i.e.* the deadbeat time itself) is determined with compromise of robustness. An example with comparison to conventional control schemes has been given to show the effectiveness of the proposed controller. Some interesting topics, such as input shaping techniques and dual-locus diagrams, are involved in this paper.

Index Terms: Dead-time compensator, deadbeat, robustness, integral process, disturbance observer

1 Introduction

In recent years, the control of processes with an integrator and dead-time has attracted much attention because of the inherent *critical stability* or *instability*. The problem originated from the fact that the well-known Smith predictor (SP) cannot be applied to these systems because of the bad disturbance response. Watanabe and Ito [1] proposed a modified Smith predictor, called *process model control* and later established as finite spectrum assignment [2, 3, 4] on the basis of the finite Laplace transform, to obtain a desired disturbance response. Astrom *et al*. [5] introduced a structure to decouple the disturbance response from the setpoint response and, hence, they can be designed separately. After that, a series of papers have been published on this subject. Zhang *et al*. [6] presented a much easier tuning formula for the control scheme in [5]. Matausek and Micic presented a modified SP by introducing a minor loop to stabilize the process with a proportional gain [7] and another scheme with a high-pass filter [8]. Normey-Rico and Camacho analyzed the robustness of a modified structure of Watanabe's in [9] and then proposed a unified approach to design a dead-time compensator for stable and integral processes with dead-time in [10]. Majhi and Atherton [11] presented a scheme for such systems as well as stable/unstable processes with dead-time. With combination of internal model control, internal model principle and Smith predictor, Stojic *et al*. proposed a control scheme to reject arbitrary disturbances with known modes [12].

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†Qing-Chang Zhong is with Dept. of Electrical & Electronic Engineering, Imperial College, Exhibition Road, London, SW7 2BT, UK. *Tel*: 0044-20-759 46295, *Fax*: 0044-20-759 46282, *Email*: zhongqc@ic.ac.uk, *URL*: http://come.to/zhongqc

This series of papers consists of three parts. In Part 1 [13], a disturbance observer-based control scheme was shown to be an effective way to control such systems. Furthermore, it was shown that the resulting disturbance response is in fact sub-ideal [14]. Although the control schemes proposed in the above cited papers are different, many of them offer the same disturbance response. This motivated the quantitative analysis of the robust stability region, the achievable specifications of the disturbance response and the stability of the controller itself in Part 2 [15]. As a result, the controller can be designed quantitatively. However, all the above-mentioned disturbance responses are of infinite impulse responses and the recovery time is very long. As shown in Part 2 [15], the sub-ideal recovery time is about $(7\beta + 2)\tau$, in which β is a control parameter with a reasonable range of $0.5 \sim 1$ and τ is the dead-time, *i.e.*, the recovery time is about $(5.5 \sim 9)\tau$. This motivated the research in Part 3 (this paper) to shorten the recovery time, even to a given time.

Deadbeat control is frequently used in discrete-time domain as well as in continuous-time domain [16, 17]. Using the idea of deadbeat control, this paper presents a design method to obtain a deadbeat disturbance response while the robust stability is still guaranteed, on the basis of the disturbance observer-based 2DOF control scheme presented in Part 1 [13]. A step disturbance is then rejected *completely in finite time*. A special feature in this scheme (more specifically, the low-pass filter) is that more than one delay elements are used in the controller unlike the common case of a Smith predictor.

The rest of the paper is organized as follows. For the reader's convenience and to render the paper self-contained, the disturbance observer-based control scheme is reviewed in Section II. In Section III the low-pass filter is designed to obtain a deadbeat disturbance response and then the control structure for implementation is derived. Control parameters are tuned in Section IV according to the robust stability. An example is given in Section V to verify the effectiveness of the controller. Finally, conclusions are made in Section VI.

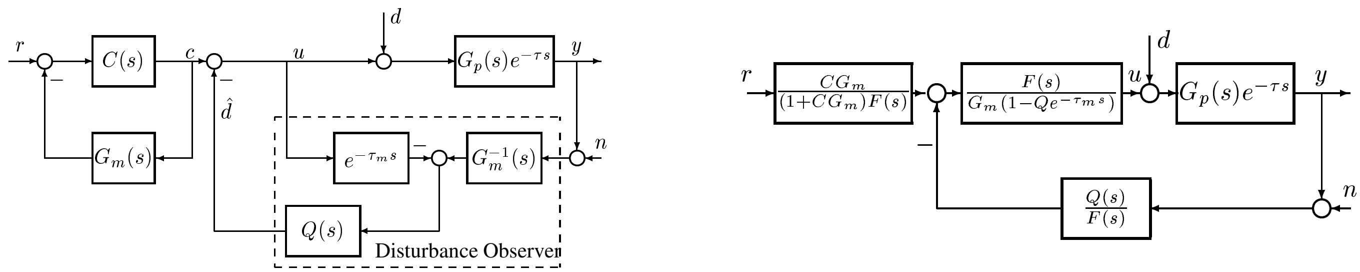
2 Disturbance Observer-based Control Scheme

Assume that an integral process with dead-time (IPDT) is described as,

$$G(s) = G_p(s)e^{-\tau s} = \frac{k}{s}G_{p0}(s)e^{-\tau s},$$

where $G_{p0}(s)$ is a stable minimum-phase transfer function with $G_{p0}(0) \neq 0$; the pure dead-time τ and static gain k are all positive.

The disturbance observer-based 2DOF control scheme [13] is revisited in Figure 1(a), where τ_m is the estimated dead-time and $G_m(s) = \frac{k}{s}$ is the nominal delay-free part of the plant. $C(s) = \frac{1}{kT}$ is designed according to the delay-free part $G_m(s)$ so that the closed-loop response meets the desired setpoint response specifications and $Q(s)$ is a low-pass filter. It has been revealed in Part 1 [13] that this scheme has many advantages. The control scheme itself shown in Figure 1(a) is *neither causal nor internally stable* because of the pure differentiator and the pole-zero cancellation between the controller and the plant. However, this structure is very good for design and understanding. Figure 1(b) provides an equivalent structure, which looks more like a common feedback system consisting of a control plant $G_p(s)e^{-\tau s}$, a measurement unit $\frac{Q(s)}{F(s)}$, an input filter $\frac{CG_m}{(1+CG_m)F(s)}$ and a controller $\frac{F(s)}{G_m(1-Qe^{-\tau_m s})}$, where $F(s)$ is a stable minimal-phase filter to make the last three blocks causal. This structure can be *causal and internally stable*



(a) The original structure

(b) An equivalent structure

Figure 1: The control structure for design

according to the internal stability theory [18]. This coincides with the fact that structures for controller design and implementation can be algebraically equivalent but analytically different[1]. In other words, algebraically equivalent schemes can have different analytical properties via different implementations. In order to avoid pole-zero cancellations between the controller and the plant, in general, it is often necessary to implement the involved finite-impulse-response (FIR) part non-dynamically for an unstable plant with dead-time [19]. The final closed-loop feedback system to be implemented is given in Figure 2 after detailed discussions of the designed controller in Section III.

In the nominal case, $G_m(s) = G_p(s) = \frac{k}{s}$ and $\tau_m = \tau$, the transfer functions from reference command r, disturbance d, and measurement noise n to output y are, respectively:

$$G_{yr}(s) = \frac{C(s)G_m(s)}{1 + C(s)G_m(s)} e^{-\tau_m s}, \tag{1}$$

$$G_{yd}(s) = G_m(s)e^{-\tau_m s}\left(1 - Q(s)e^{-\tau_m s}\right), \tag{2}$$

$$G_{yn}(s) = Q(s)e^{-\tau_m s}. \tag{3}$$

As can be seen, the Smith principle, *i.e.* the dead-time is not included in the characteristic equation, is satisfied. Moreover, for a low-pass filter $Q(s)$, $G_{yd}(s) \to 0$ at low frequencies and $G_{yn}(s) \to 0$ at high frequencies. Hence, the system has good performance to reject disturbances and measurement noises. More importantly, $G_{yr}(s)$ and $G_{yd}(s)$ have been decoupled because of the 2DOF structure; the setpoint response is determined by $C(s)$ while the disturbance response is determined by $Q(s)$. In order to guarantee the closed-loop stability, the condition $Q(0) = 1$ must be satisfied as can be seen from (2).

Substitute $C(s) = \frac{1}{kT}$ into (1), the following setpoint response is obtained:

$$G_{yr}(s) = \frac{1}{Ts + 1} e^{-\tau_m s}. \tag{4}$$

The achievable specifications can be easily evaluated. In later sections, attention will be paid to the design of the low-pass filter $Q(s)$ to obtain a dead-beat disturbance response.

[1] See, Zhou K., Feedback Control System, lecture notes, pp.51, available from http://www.ece.lsu.edu/kemin/class/ee4580/fdc.pdf. Accessed on 18/04/02.

3 Deadbeat Disturbance Response

3.1 Design of $Q(s)$

Lemma 1. [16, 17] *Let $a(s)$ and $b_i(s)$ ($i = 0, 1, \cdots, q$) be polynomials in s and $\frac{b_i(s)}{a(s)}$ strictly proper in s. $b(s)$ and $\phi(s)$ are given by*

$$b(s) = b_0(s) + b_1(s)e^{-sT_1} + \cdots + b_q(s)e^{-sT_q} \ \ and \ \ \phi(s) = \frac{b(s)}{a(s)},$$

where $T_q > T_{q-1} > \cdots > T_1 > 0$. Let a_j ($j = 1, \cdots, p$) denote the zeros of $a(s)$ with multiplicity r_j. Assume $b(s)$ satisfies

$$\left.\frac{d^l}{ds^l}b(s)\right|_{s=a_j} = 0 \qquad (l = 0, 1, \cdots, r_j - 1;\ j = 1, \cdots, p), \tag{5}$$

then, $\phi(t) = \mathcal{L}^{-1}[\phi(s)]$ settles to zero in finite time $t = T_q$, i.e. $\phi(t) = \begin{cases} \phi_0(t) & (0 \le t < T_q) \\ 0 & (t \ge T_q) \end{cases}$.

Remark 3.1. This means that all the zeros of the denominator are also the zeros of the numerator. Such a $\phi(s)$ is called a pseudo-differential polynomial [20, 4] because it is actually an entire function of s but not in the exact form of a polynomial.

Remark 3.2. $b(s)$ has infinite zeros because of the multiple delay elements, in addition to the zeros of $a(s)$.

Theorem 1. *The disturbance observer-based control system for an integral process with dead-time rejects a step disturbance in finite time $t = 2\tau_m + T_2$ if the low-pass filter is chosen as*

$$Q(s) = \frac{1 - q_1(1 - e^{-T_1 s}) - q_2(1 - e^{-T_2 s})}{\lambda s + 1} \tag{6}$$

with

$$\begin{cases} q_1 = -\frac{(e^{T_2/\lambda} - 1)(\lambda + \tau_m) - T_2}{T_2 - T_1 + T_1 e^{T_2/\lambda} - T_2 e^{T_1/\lambda}} \\ q_2 = \frac{(e^{T_1/\lambda} - 1)(\lambda + \tau_m) - T_1}{T_2 - T_1 + T_1 e^{T_2/\lambda} - T_2 e^{T_1/\lambda}} \end{cases} \tag{7}$$

where $T_2 > T_1 > 0$, and $\lambda > 0$ is a free parameter.

Proof. The denominator polynomial of a step disturbance is $D(s) = s$. According to (2), the disturbance response is deadbeat if $Q(s)$ is chosen such that the following transfer function

$$\phi(s) = G_m(s)\left(1 - Q(s)e^{-\tau_m s}\right)e^{-\tau_m s} \cdot \frac{1}{D(s)} \tag{8}$$

satisfies Lemma 1. $Q(s)$ in (6) is such a candidate. Substitute (6) into (8), then

$$\phi(s) = \frac{\lambda s + 1 - e^{-\tau_m s} + q_1(1 - e^{-T_1 s})e^{-\tau_m s} + q_2(1 - e^{-T_2 s})e^{-\tau_m s}}{s^2(\lambda s + 1)} k e^{-\tau_m s}. \tag{9}$$

The poles of $\phi(s)$ are $0,\ 0$ and $-\frac{1}{\lambda}$; $\phi(s)$ also has a zero at $s=0$. Hence, in order to make $\phi(s)$ be a pseudo-differential polynomial, the following equations should be met according to Lemma 1:

$$\begin{cases} -q_1T_1 \ - \ q_2T_2 - \lambda = \tau_m \\ 1 - q_1(1-e^{T_1/\lambda}) - q_2(1-e^{T_2/\lambda}) = 0 \end{cases} . \tag{10}$$

For $T_2 > T_1 \ > 0$, the solutions of q_1 and q_2 can be obtained as (7). This completes the proof. □

Remark 3.3. $Q(s)$ may be interpreted as a low-pass filter, $\frac{1}{\lambda s+1}$, cascaded by an *input shaper* [21, 22, 23], $1 - q_1 - q_2 + q_1e^{-T_1s} + q_2e^{-T_2s}$.

Remark 3.4. For arbitrary disturbance polynomial $D(s)$, it is possible to obtain a deadbeat response provided that the number of the delay elements is enough so that $\phi(s)$ in (8) is a pseudo-differential polynomial.

Remark 3.5. For any tuning parameters $T_2 > T_1 \ > 0$ and $\lambda > 0$, the static gain of $Q(s)$ given in (6) is always equal to 1. This guarantees the stability of the closed-loop system.

3.2 Implementation of the controller

In order to make the measurement unit $\frac{Q(s)}{F(s)}$, the input filter $\frac{CG_m}{(1+CG_m)F(s)}$ and the controller $\frac{F(s)}{G_m(1-Qe^{-\tau_m s})}$ in Figure 1(b) *causal*, the relative degree of $F(s)$ has to be 1 for the above design. The simplest $F(s)$ is to choose

$$F(s) = \frac{1}{\lambda s + 1},$$

without introducing a new parameter. Then,

$$\frac{CG_m}{(1+CG_m)F(s)} = \frac{\lambda s+1}{Ts+1} \quad \text{and} \quad \frac{Q(s)}{F(s)} = 1 - q_1(1-e^{-T_1s}) - q_2(1-e^{-T_2s}),$$

and

$$\begin{aligned} \frac{F(s)}{G_m(1-Qe^{-\tau_m s})} &= \frac{s}{k(\lambda s + 1 - e^{-\tau_m s} + q_1(1-e^{-T_1s})e^{-\tau_m s} + q_2(1-e^{-T_2s})e^{-\tau_m s})} \\ &= \frac{1}{k}\cdot\frac{1}{\lambda + \frac{1-e^{-\tau_m s}}{s} + q_1\frac{1-e^{-T_1s}}{s}e^{-\tau_m s} + q_2\frac{1-e^{-T_2s}}{s}e^{-\tau_m s}} \\ &= \frac{1}{k}\cdot\frac{\frac{1}{\lambda}}{1+\frac{1}{\lambda}Z(s)}, \end{aligned}$$

where

$$Z(s) \doteq \frac{1-e^{-\tau_m s}}{s} + q_1\frac{1-e^{-T_1s}}{s}e^{-\tau_m s} + q_2\frac{1-e^{-T_2s}}{s}e^{-\tau_m s}$$

is an FIR block or a pseudo-differential polynomial. Hence, the control system can now be implemented as the structure shown in Figure 2, where the FIR block $Z(s)$ has to be implemented in a non-dynamical way [19], *i.e.* to implement the signal v in

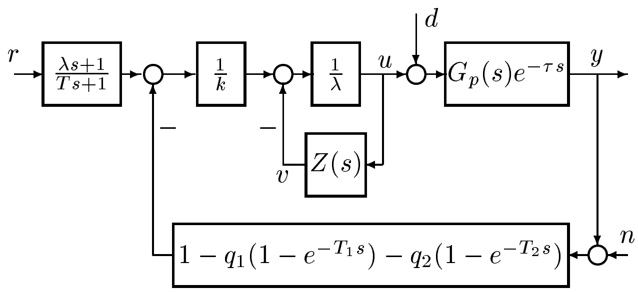


Figure 2: The control structure for implementation

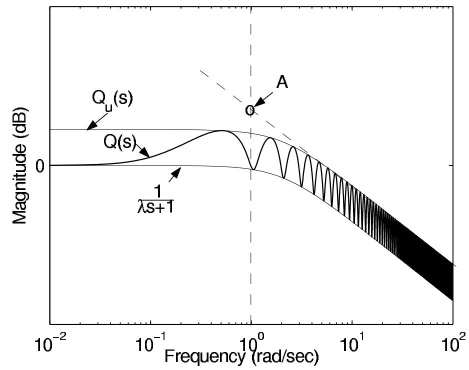


Figure 3: Magnitude frequency response of $Q(s)$

time-domain as:

$$v(t) = \int_{t-\tau_m}^{t} u(t)dt + q_1 \int_{t-\tau_m-T_1}^{t-\tau_m} u(t)dt + q_2 \int_{t-\tau_m-T_2}^{t-\tau_m} u(t)dt.$$

By doing so, $Z(s)$ does not possess any pole, even at $s = 0$, and there does not exist any unstable pole-zero cancellation between the controller and the plant. Hence, the control scheme implemented in Figure 2 is internally stable.

In this control structure, the disturbance response from d to y, considering uncertainties in the plant, can be represented as

$$\begin{aligned} G_{yd}(s) &= \frac{\frac{k}{s} G_{p0}(s)e^{-\tau s}}{1 + (1 - q_1(1-e^{-T_1 s}) - q_2(1-e^{-T_2 s}))\frac{1}{\lambda+Z(s)}\frac{1}{s}G_{p0}(s)e^{-\tau s}} \\ &= \frac{k(\lambda + Z(s))G_{p0}(s)e^{-\tau s}}{\lambda s + sZ(s) + (1 - q_1(1-e^{-T_1 s}) - q_2(1-e^{-T_2 s}))G_{p0}(s)e^{-\tau s}}. \end{aligned}$$

Since $G_{p0}(0) \neq 0$ and $Z(s)$ does not have a pole, $G_{yd}(s)$ does *not* have an unstable pole at $s = 0$. In the nominal case, where $G_{p0}(s) = 1$ and $\tau_m = \tau$, the disturbance response is

$$G_{yd}(s) = \frac{k\,(\lambda + Z(s))\,e^{-\tau s}}{\lambda s + 1}. \tag{11}$$

There is only one pole at $s = -\frac{1}{\lambda}$ (since $Z(s)$ does not possess a pole) and the nominal system is stable. Moreover, this (stable) pole is cancelled by the zero at $s = -\frac{1}{\lambda}$ since $\lambda + Z(-\frac{1}{\lambda}) = 0$, according to (10). This confirms that the obtained disturbance response is indeed deadbeat in the nominal case.

4 Parameter Tuning and Robustness

The complementary transfer function of the nominal system is

$$T(s) = Q(s)e^{-\tau_m s}.$$

Hence, for a multiplicative uncertainty $\Delta(s) \in H_\infty$, the system is robustly stable if $\|Q(s)\|_\infty < \frac{1}{\|\Delta(s)\|_\infty}$, *i.e.*, the magnitude frequency response $|Q(j\omega)|$, of which a typical example is shown in Figure 3, stays under $\frac{1}{|\Delta(j\omega)|}$.

It can be shown that for $T_2 > T_1 > 0$ and $\lambda > 0$, the (delay) gains satisfy

$$1 - q_1 - q_2 > 0,\ q_1 < 0 \text{ and } q_2 > 0. \tag{12}$$

The upper envelope of $Q(s)$ given in (6) is,

$$Q_u(s) = \frac{|1 - q_1 - q_2| + |q_1| + |q_2|}{\lambda s + 1} = \frac{1 - 2q_1}{\lambda s + 1}.$$

Hence, the following theorem holds:

Theorem 2. *The system described in Theorem 1 is robustly stable if*

$$1 - 2q_1 < \frac{1}{\left\| \frac{\Delta(s)}{\lambda s + 1} \right\|_\infty},$$

for a multiplicative uncertainty $\Delta(s) \in H_\infty$.

The high frequency asymptote of the upper envelope $Q_u(s)$ is

$$\log |Q_u^a(j\omega)| = \log \frac{1 - 2q_1}{\lambda} - \log \omega.$$

It intersects with the straight line $\omega = 1$rad/sec at $\log \frac{1-2q_1}{\lambda}$, noted as A in Figure 3. $1 - 2q_1$ can be interpreted as the *cost of robustness to obtain a deadbeat disturbance response* and $J = \frac{1-2q_1}{\lambda}$ as *a robustness indicator*. For given T_2 and λ, T_1 can be chosen to make point A as low as possible, *i.e.*

$$\min_{T_1} J = \min_{T_1} \frac{1 - 2q_1}{\lambda}$$

to obtain the largest possible robust stability region. With consideration of (7) and (12), the robustness indicator can be simplified as

$$J = \frac{1}{\lambda} \left(1 + \frac{2(\lambda + \tau_m)(e^{T_2/\lambda} - 1) - 2T_2}{T_2 - T_1 + T_1 e^{T_2/\lambda} - T_2 e^{T_1/\lambda}} \right). \tag{13}$$

Since $(\lambda + \tau_m)(e^{T_2/\lambda} - 1) - T_2 > 0$ and $T_2 - T_1 + T_1 e^{T_2/\lambda} - T_2 e^{T_1/\lambda} > 0$ for $T_2 > T_1 > 0$ and $\lambda > 0$, J is always larger than $\frac{1}{\lambda}$. Differentiate J with respect to T_1 and let it be 0, then

$$-1 + e^{T_2/\lambda} - \frac{T_2}{\lambda} e^{T_1/\lambda} = 0,$$

which offers

$$T_1 = \lambda \ln \frac{e^{T_2/\lambda} - 1}{T_2/\lambda}, \tag{14}$$

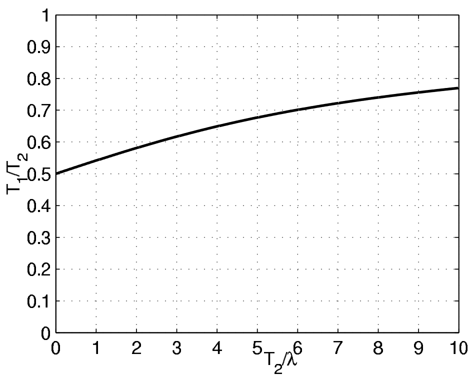


Figure 4: The optimal ratio T_1/T_2 with respect to T_2/λ

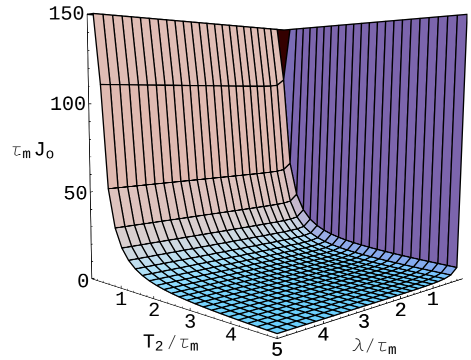


Figure 5: Scaled robustness indicator $\tau_m J_o$

or

$$\frac{T_1}{T_2} = \frac{\lambda}{T_2}\ln\frac{e^{T_2/\lambda}-1}{T_2/\lambda}.$$

A graphical representation of this function is shown in Figure 4. If $T_2/\lambda \to 0$, then $T_1 \to 0.5T_2$; If $T_2/\lambda \to \infty$, then $T_1 \to T_2$. Thus, T_1 is always less than T_2. The corresponding minimal J is then obtained as

$$J_o = \frac{1}{\lambda}\left(1 + \frac{2(1+\frac{\tau_m}{\lambda})(e^{T_2/\lambda}-1) - 2\frac{T_2}{\lambda}}{\frac{T_2}{\lambda} + (e^{T_2/\lambda}-1)\left(\ln\frac{e^{T_2/\lambda}-1}{T_2/\lambda} - 1\right)}\right). \tag{15}$$

A scaled J_o by τ_m is shown in Figure 5 with respect to $\frac{T_2}{\tau_m}$ and $\frac{\lambda}{\tau_m}$.

Remark 4.1. As can be seen, the larger T_2 and λ the better the robustness. The robustness becomes worse drastically for $T_2 < \tau_m$ or very small λ/τ_m. Hence, T_2/τ_m or λ/τ_m cannot be chosen too small, although a small λ is helpful for the dynamics of the disturbance response as given in (11).

Remark 4.2. If $\lambda \to +\infty$, then $\tau_m J_o \to \frac{16+8\frac{T_2}{\tau_m}}{(\frac{T_2}{\tau_m})^2}$. In fact, for $\lambda > \tau_m$, the robustness indicator J_o varies slightly. This means that λ can no longer be used as a fine tuning parameter to meet the robustness as in a common disturbance observer-based control scheme using a rational low-pass filter [13, 24]. A reasonable value of λ is $(0.5 \sim 1)\tau_m$.

Remark 4.3. If $T_2 \to +\infty$, then $J_o \to \frac{1}{\lambda}$; the system degenerates into a common system without the property of a deadbeat disturbance response and so the robustness. This means that T_2 has to be used to compromise between the deadbeat time and the robustness. If a deadbeat disturbance response is desired, the robustness has to be sacrificed to some extent. If a good robustness is desired, the deadbeat time cannot be chosen too short. This also indicates that delay T_2 takes the place of the bandwidth of the low-pass filter to compromise between the robustness and the disturbance response. This compromise can never be overcome.

Remark 4.4. For given T_2 and λ, J_o is proportional to the dead-time τ_m as shown in (13). This means that a system having a longer dead-time has to pay more for the robustness in order to obtain a deadbeat disturbance response.

In summary, the parameters are determined as follows:

Step 1. Initially choose $T_2 = (1.5 \sim 2)\tau_m$, then the recovery time is $(3.5 \sim 4)\tau_m$, which is much faster than that of the sub-ideal disturbance response $(5.5 \sim 9)\tau_m$ (if the robustness is met);

Step 2. Choose $\lambda = (0.5 \sim 1)\tau_m$, calculate T_1 from (14) and then q_1 and q_2 from (7).

Step 3. Evaluate the robustness. If the robust stability condition is not met, then increase λ and repeat Step 2 to check if the robust stability condition can be slightly over-met; if the robust stability condition is much over-met, then decrease T_2 and repeat Step 2 till the robust stability condition is slightly over-met. If increasing λ cannot meet the robust stability condition, then increase T_2 and repeat the above procedures again till the robust stability condition is slightly over-met.

5 An example

Consider the following widely studied integral process with dead-time [5, 8, 9]:

$$G_m(s) = \frac{1}{s}, \tau_m = 5\,\text{sec},$$

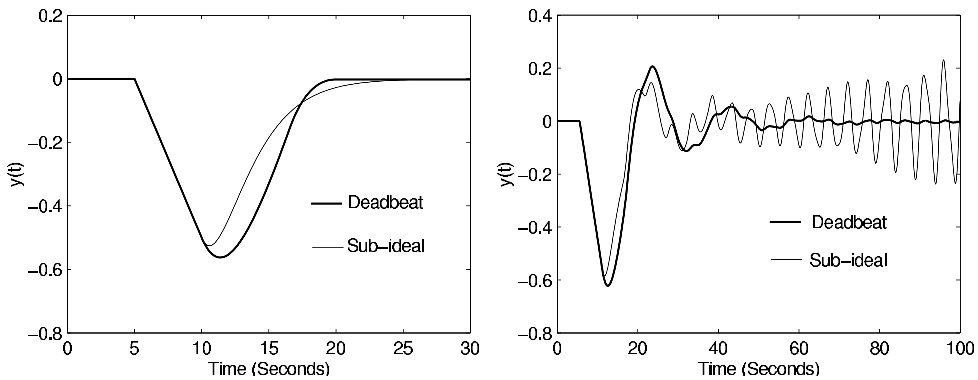
assuming that the worst multiplicative uncertainty is $\Delta(s) = \frac{1}{0.1s+1}e^{-0.5s} - 1$.

Choose $T_2 = 2\tau_m = 10$sec and $\lambda = 0.5\tau_m = 2.5$sec, then $T_1 = 6.5$sec, $q_1 = -1.75$, $q_2 = 0.39$. The disturbance response of $d(t) = -0.1$ in the nominal case is shown in Figure 6(a) and the one in the worst case of uncertainty is shown in Figure 6(b). The nominal disturbance response is deadbeat at $t = 2\tau_m + T_2 = 20$sec and the response in the worst case uncertainty becomes worse but still very good. As mentioned in the Introduction, many other different control schemes offer the same disturbance response, where the recovery time of the disturbance response is very long. A comparison to the disturbance response obtained by those schemes is given as follows: the recovery time is designed to be very close to $t = 20$sec using a low-pass filter $\frac{9s+1}{(2s+1)^2}$ for the conventional control schemes, e.g. the one studied in Part 1 [13], to obtain a sub-ideal disturbance response. The corresponding disturbance responses in the nominal and worst cases are noted as Sub-ideal in Figure 6 (a) and (b), respectively. As can be seen, the system becomes unstable in the worst case of uncertainty. In other words, in order to obtain the same recovery time, the proposed deadbeat control scheme has much better robustness than the conventional scheme. In order to obtain similar robustness, the recovery time obtained by the conventional scheme is much longer. The advantage of the conventional schemes is a relatively smaller dynamic error.

The following analysis shows that the controller itself has no right-half plane pole and hence is stable. The characteristic equation of the controller itself, $1 - Q(s)e^{-\tau_m s} = 0$, is equivalent to

$$\lambda s + 1 = (1 - q_1 - q_2)e^{-\tau_m s} + q_1 e^{-(T_1+\tau_m)s} + q_2 e^{-(T_2+\tau_m)s}.$$

Denote the left side as $L_1(s)$ and the right side as $L_2(s)$, of which the loci are shown in Figure 7 for $\omega = 0 \to +\infty$. Locus $L_1(j\omega)$ is a straight line from $(1,\ j0)$ to $(1,\ j\infty)$ and locus $L_2(j\omega)$ lies inside the circle at $(0, 0)$ with a radius of $1 - 2q_1 = 4.5$. $L_1(j\omega)$ intersects with the circle at $\omega_0 = \frac{\sqrt{(1-2q_1)^2-1}}{\lambda} = 1.75$rad/sec. For $\omega > \omega_0$, the two loci do not intersect. For $0 < \omega < \omega_0$ there is



(a) The nominal case (b) The worst case of uncertainty

Figure 6: Comparison of disturbance responses

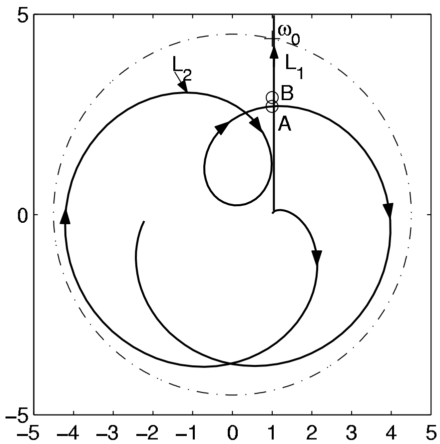


Figure 7: Dual-locus diagram of the controller

only one intersection (noted as A in Figure 7), of which the corresponding frequency of $L_2(j\omega)$ is about $\omega_A = 1.165$rad/sec. For the same frequency, $L_1(j\omega_A)$ arrives at point B. Hence, L_1 arrives at the intersection A earlier than L_2. According to the dual-locus diagram [25, 26], no right-half plane pole exists in the controller and, hence, the controller itself is stable.

6 Conclusions

Using multiple delay elements, this paper presents a control scheme to obtain a deadbeat response under a step disturbance for integral processes with dead-time. For the same robustness, the proposed control scheme obtains faster disturbance response; for the same recovery time, the proposed control scheme obtains better robustness. A special feature of the controller is that two delay elements are intentionally used in the controller but only one delay element is used in a common Smith predictor. Moreover, the delays are tuning parameters rather than a fixed one equal to the nominal dead time. The longer delay (*i.e.* the deadbeat time) is tuned with compromise of robustness and the shorter delay is optimally determined to minimize the robustness indicator and, hence, to maximize the allowable uncertainty bound. With consideration of the internal model principle, the controller is potential to reject arbitrary disturbances [12]. Some interesting topics, such as input shaping techniques and dual-locus diagrams, have been involved in this paper.

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