On the Optimisation of the Longitudinal Location of the Mass Centre of a Formula One Car for two Circuits

Daniele Casanova[\$], Robin S. Sharp* and Pat Symonds[\$]
***School of Engineering, Cranfield University, Bedford, MK43 0AL, United Kingdom**
[\$]Renault F1 Team Ltd, Enstone, Oxford, OX7 4EE, United Kingdom

Whiteways Technical Centre
Enstone, Chipping Norton, OX7 4EE, United Kingdom
Phone: +44 +1608 678000
Fax: +44 +1608 678514
E-mail: Daniele.Casanova@uk.renaultf1.com

In previous work, a method based in non-linear programming for finding the minimum possible lap time for a given "virtual" car on a given "virtual" circuit has been described. Results have demonstrated the repeatability and accuracy of the process. The modelling, simulation and optimisation scheme has been applied to finding the fastest lap times for the Formula One circuits in Barcelona and Suzuka for a representative car with wide variations in the longitudinal positioning of the mass centre. Results show the increasing difficulty of solving the simulation/optimisation problem as the rearward weight bias increases, changes in the optimal control strategies as the car front to rear balance changes, tyre shear force utilisation factors in the different cases and the potential advantage of cars with rearward mass centre locations. Comparative results are given and the optimised behaviour fully discussed.

Keywords/Vehicle Dynamics, Modelling, Simulation, Optimisation.

1. INTRODUCTION

The longitudinal position of the mass centre is one of the primary set-up parameters for a Formula One racing car. With the current engine and chassis technology, engineers are able to design cars which are significantly below the minimum weight set by the sporting regulation, requiring a considerable amount of ballast for the car in running trim. The position of the ballast may be varied in order to adjust the longitudinal mass centre location typically by 2 % to 4 % of the wheelbase. Ideally, in order to obtain the maximum possible performance, the weight distribution should be adjusted in such a way that the tyre force capacity is used as closely as possible to its maximum in all conditions during a lap. For example, one may expect that in steady state cornering conditions the maximum speed is achieved when both front and rear axle tyre lateral forces are used at their 100 % limit. However, this condition would not be ideal for the controllability of the vehicle. In order to guarantee sufficient stability for the driver to control the car it is imperative to configure it in such a way that the rear tyres maintain some spare force capacity in all conditions. Furthermore, very seldom is a racing car in pure steady state cornering conditions. The longitudinal weight distribution has obviously an effect on the balance of the car during braking and turning into a corner, as well as an influence on traction when driving out of a corner. Ultimately, the optimal weight distribution will vary depending on the circuit geometry, e.g. a slow, twisty circuit may favour a rearward weight distribution for traction, while a fast circuit may favour a more forward biased weight to enhance stability.

Lap simulation techniques are nowadays a fundamental aid to study and optimise the set-up of a racing car. In this work a dynamic lap simulation program is used to investigate the effect of the weight distribution on the theoretical minimum possible lap time for two different circuits. Starting from a baseline car set-up, nine different longitudinal positions of the mass centre have been considered. The trend in lap time changes is clearly shown and the different driving strategies required for driving the different car configurations are highlighted with the aid of the dynamic lap simulation.

2. THE SIMULATION PROGRAM

In a previous work, different mathematical models for the prediction of the best lap time of a circuit racing car were described [1]. In the most common approach, which is widely used today, the car performance envelope is represented by means of the "*g/g/speed*" surface [2]. This is a set of planar "*g/g*" diagrams representing the maximum lateral and longitudinal accelerations that the vehicle can achieve in steady state conditions across its operative speed range. The lap simulation technique requires the racing line to be defined by some means. Then, the path around the circuit is divided in arbitrarily short segments with constant curvature and the maximum speed attainable in each segment computed. The speed is limited either by the lateral acceleration limit for the segment's curvature or by the longitudinal acceleration limit when moving from one segment to the next [3,4].

A novel approach for the simulation of the performance of a racing car describes the problem as one of Optimal Control [5]. For a datum vehicle and circuit model, the task is to compute the optimal vehicle controls, i.e. the steer angle and the driving/braking torque, which allow the virtual car to be "driven" around the circuit in as short a time as possible, with the sole constraint of remaining within the road boundaries. The primary advantage of this method is that the transient behaviour of the racing car is taken into account in the lap simulation. Furthermore, the driving strategy is not constrained by imposing the racing line, which is computed in the process instead. This is advantageous for the accuracy of the lap time predictions as the optimal driving strategy is different for different car configurations.

Relevant Optimal Control approaches were reviewed in [1]. In the same work the present authors proposed the use of a direct method, i.e. the *parallel shooting method* [6], for the solution of the minimum time problem. This approach has subsequently been developed into a complete, dynamic lap simulation program. Results have been presented in [7,8] and the repeatability and accuracy of the process have been demonstrated.

In general terms, the dynamic lap simulation technique can be described as a learning process. For a given set of controls, the simulator evaluates the lap time and the car positions along the circuit by solving the equations of motion. Next, the program evaluates the sensitivity of the computed lap time and of the constraint information with respect to the control inputs. This can be accomplished directly since the ideal continuous problem is reduced to a finite-dimensional problem by discretising the controls over a grid of fixed points distributed along the circuit, as was described in [8]. Finally, the optimisation algorithm uses the sensitivity information to improve the control inputs in order to minimise the lap time and satisfy the problem's constraints. This cycle is repeated until convergence is achieved to specified tolerances. The evaluation of the sensitivities can be seen as the "learning step", and is the most computationally intensive part of the process. Furthermore, ensuring good accuracy for all partial derivatives is crucial for the optimisation algorithm to work robustly.

A considerable improvement in computational speed and accuracy when evaluating derivatives can be achieved by using Automatic Differentiation [9]. Automatic Differentiation is a programming technique for obtaining derivatives of numerical functions without the labour of deriving explicit symbolic expressions. A mathematical program is augmented by associating to each algebraic operator its corresponding derivative operations. For example, for the power function operator "x^n" the program will be instructed to perform the evaluation of "$n{\cdot}x^{n-1}$" on the variable which carries forwards the values of the derivatives. When evaluating the function, the chain rule is repeatedly applied and partial derivatives are obtained to the same order of accuracy as the function evaluation. The application of Automatic Differentiation to the minimum time vehicle manoeuvring optimisation problem was presented in [10], and the results showed an increase in computational speed by up to ten times. Also the ability to converge to tighter tolerance was demonstrated, reflecting the enhanced precision of the computed derivatives.

3. VEHICLE AND CIRCUIT MODELS

The vehicle is represented as having seven degrees of freedom. The chassis is treated as a rigid body with three degrees of freedom, the yaw angle and the lateral and longitudinal displacements. The wheels each have a spin degree of freedom relative to the body. The chassis model includes the roll axis position, the roll stiffness distribution, the mass centre height and the track width. These features are sufficient to evaluate a quasi-steady-state approximation of the lateral and longitudinal load transfers, giving realistic wheel loads.

A simple representation of the aerodynamic forces is employed by assuming constant drag and lift coefficients. The aerodynamic drag is applied at the height of the vehicle centre of gravity. The centre of application of the aerodynamic lift is the same for all speeds and is determined by specifying the down force distribution between the front and the rear axles.

The tyre lateral and longitudinal forces are introduced using the Magic Formula Tyre Model which features the use of weighting functions to account for combined slip conditions [11]. Static wheel camber and toe angle settings are also accounted for.

The vehicle lateral control variable is the steer angle applied to the front wheels. A parallel steer geometry is considered for the steering system A single control variable is defined for the longitudinal control. It is assumed that this variable represents the throttle aperture when it is positive, or a fraction of the maximum braking torque available when it is negative.

The drive train is modelled using a steady-state engine torque map, function of engine rotational velocity and throttle demand. The driving torque is transferred to the rear wheels through a six ratios gearbox and a limited slip differential. The braking torque is applied to all four wheels and is shared among the front and rear axles using constant coefficients. The engine brake effect is then added to the rear axle.

The model of a circuit is essentially described by the following (non-independent) parameters, see Fig. 1:

- The co-ordinates of its centre line in a reference axes system fixed in space, x_t, y_t;
- The local radius r_t of the road centre line;
- The tangent angle of the centre line, ψ_t;
- The road width, w_t.

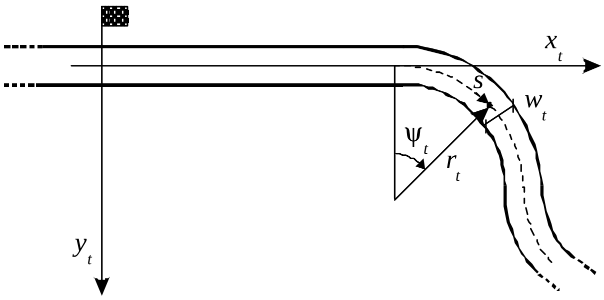

Fig. 1 Circuit model description.

These data are expressed as functions of the independent path co-ordinate s, i.e. the distance travelled along the road centre line from the start-finish line. Additional information may be supplied, e.g. road elevation, road camber angle, variations in the friction coefficient of the road surface, etc.

The dynamic lap simulation program requires further information regarding the discretisation of the state and control variables, and the initial vehicle trajectory and controls. These data are conveniently decided when modelling the circuit and are included as a part of the track data file. The criteria for defining the proper problem discretisation were extensively described in [7,8].

4. RESULTS

As was anticipated earlier, the best theoretical lap times for Barcelona and Suzuka have been computed for nine different longitudinal positions of the vehicle mass centre. Each case has been repeated four times by starting the lap simulation from different initial trial solutions. This is aimed to test the repeatability of the solution yielded by the iterative solver.

Table 1 summarises the results. For each case the best and the worst lap times obtained from the four runs have been reported. On average the gap between best and worst solutions is of the order of one tenth of a second. Also the average number of iterations taken by the solver is reported. The rows highlighted in bold refer to the vehicle baseline configuration.

The variation of lap times against the vehicle weight distribution is represented in Figs 2 and 3. For both circuits, the results indicate that moving the centre of mass considerably rearwards yields significantly faster lap times. The figures also show the average number of iterations necessary for solving each case. It is very interesting to observe the trend, that is, the more the vehicle becomes oversteer biased, the more difficult it is for the optimisation program to converge. When an oversteer biased vehicle is close to its lateral limit, it is very likely to spin as a result of small changes applied to the control inputs. In the case of the dynamic lap simulation, this results in numerical problems as certain states such as the vehicle yaw rate and the lateral velocity become very sensitive to small changes in steer and throttle/brake controls. The greater difficulty which arises in solving the problem can be compensated by refining the discretisation scheme, as was explained in [7,8].

Fig. 4 shows a comparison between the lap speeds achieved by the slowest and the fastest car configurations for the two circuits. The car with a more rear biased weight distribution gains speed in all corners. The gain is greater for the faster corners, where one may expect that the greater aerodynamic downforce enhances the benefit of maximising the utilisation of the tyre forces.

Table 1 Simulation results summary

Barcelona			
Weight % front/rear	No. of iter.	Best lap time	Worst lap time
46.6/53.4	137	1’ 21” 347	1’ 21” 476
45/55	118	1’ 21” 113	1’ 21” 199
43.5/56.5	**173**	**1’ 20” 714**	**1’ 20” 871**
42/58	146	1’ 20” 510	1’ 20” 660
40.5/59.5	176	1’ 20” 303	1’ 20” 401
38.9/61.1	200	1’ 20” 149	1’ 20” 186
37.4/62.6	200	1’ 20” 177	1’ 20” 185
35.9/64.1	198	1’ 20” 259	1’ 20” 356
34.3/65.7	200	1’ 20” 330	1’ 20” 423
Suzuka			
Weight % front/rear	No. of iter.	Best lap time	Worst lap time
47.1/52.9	114	1' 36" 826	1' 37" 034
45.5/54.5	87	1' 36" 560	1' 36" 808
44/56	**86**	**1' 36" 145**	**1' 36" 305**
42.5/57.5	188	1' 35" 769	1' 35" 871
40.9/59.1	159	1' 35" 530	1' 35" 686
39.4/60.6	190	1' 35" 336	1' 35" 468
37.9/62.1	190	1' 35" 274	1' 35" 394
36.4/63.6	200	1' 35" 364	1' 35" 557
34.8/65.2	200	1' 35" 396	1' 35" 641

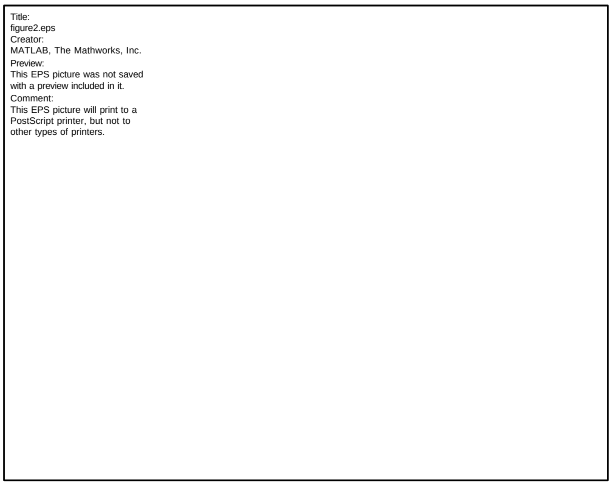

Fig. 2 Lap time and average iterations vs. weight distribution, Barcelona.

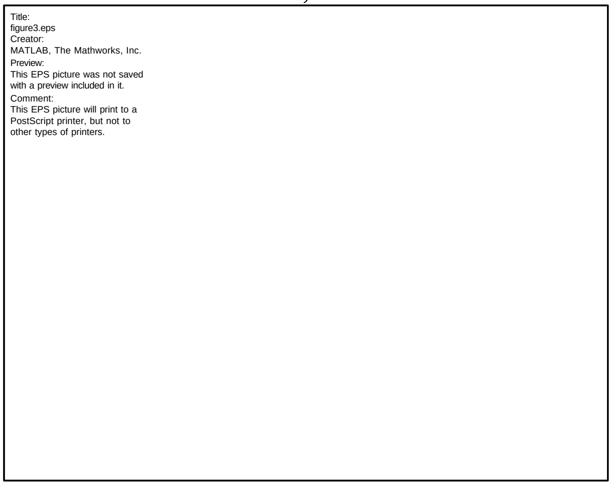

Fig. 3 Lap time and average iterations vs. weight distribution, Suzuka.

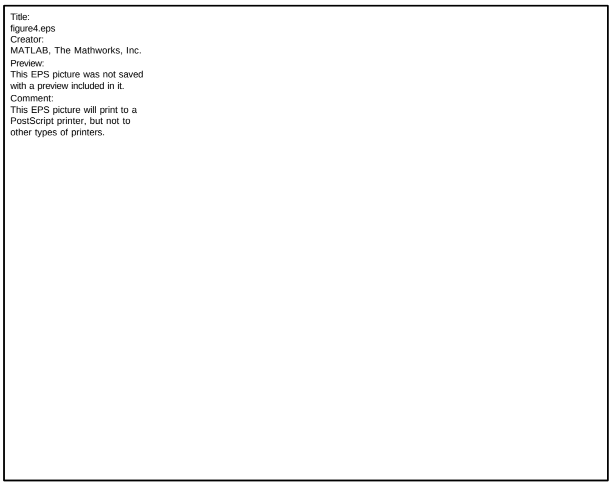

Fig.4 Vehicle longitudinal velocity comparison, fastest car vs. slowest car for the two circuits.

It is possible to quantify how vigorously the front and rear tyres are used during a lap for the different car configurations. At any position along the circuit each tyre generates a lateral force F_y which, in combined slip conditions, is a function of the slip angle a, the slip ratio k, the vertical load F_z and the camber angle g.

By using the tyre model equations we can evaluate the maximum lateral force limit in that particular condition:

$$F_{y\_MAX} = \max_{a} \left(F_y \left(a, k, F_z, g \right) \right)$$
$$at\ constant\ k, F_z, ?\ , varying\ a$$
(1)

We may then define the tyre lateral saturation as the ratio between the actual force and the maximum possible force:

$$Lat\_Sat = \frac{F_y}{F_{y\_MAX}} \times 100 \qquad [\%] \qquad (2)$$

The tyre lateral saturation index is then computed for the whole lap by post-processing the simulation results, and it will vary from 0, when the car is driving along a straight line, up to 100 % for either the front or the rear tyres during cornering, depending on the weight distribution. In order to compare different car configurations, we may reduce the measure of the tyre lateral force usage over the lap distance S to a single number by simply considering the mean integral of the lateral saturation:

$$Average\_tyre\_usage = \frac{1}{S} \int_0^S Lat\_Sat(s)\, ds$$
(3)

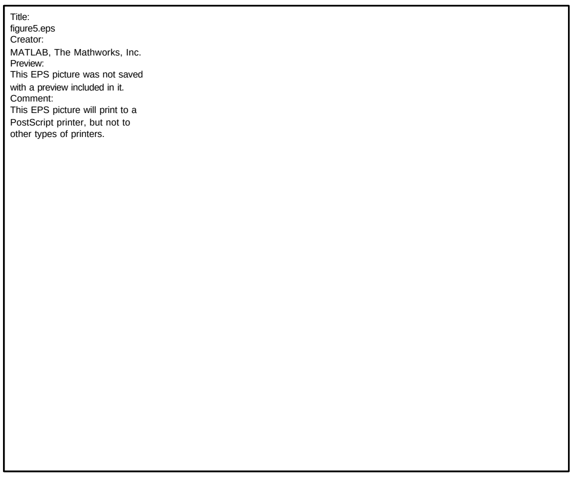

Fig. 5 Average tyre lateral force utilisation , Barcelona.

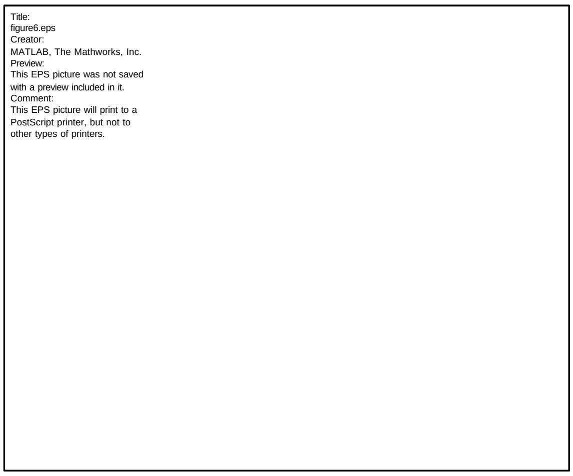

Fig. 6 Average tyre lateral force utilisation, Suzuka.

Figs 5 and 6 show the average tyre lateral force usage over a lap for Barcelona and Suzuka respectively. In each figure three cases have been considered. The left bar refers to the car with the most forward weight bias, which is also the slowest configuration for both circuits. The middle bar refers to the optimal configuration which yields the best lap time. The third bar refers to the car with the most rearward weight bias. The trend for the case of Barcelona is very clear. By moving the centre of mass towards the rear of the car, the tyre utilisation index for the front tyres decreases steadily, while that for the rear tyres increases. Interestingly, the optimal configuration, which yields the best lap time, is also that where the front and rear tyres are used equally over the lap. For the case of Suzuka the results are essentially the same, with the exception that the front tyre utilisation increases slightly for the third case. This, however, may have to do with some residual noise in the solution as convergence was more difficult for the most rearward weight distribution, as explained earlier. In any case the front tyre utilisation for the third case is still lower than that of the rear tyres.

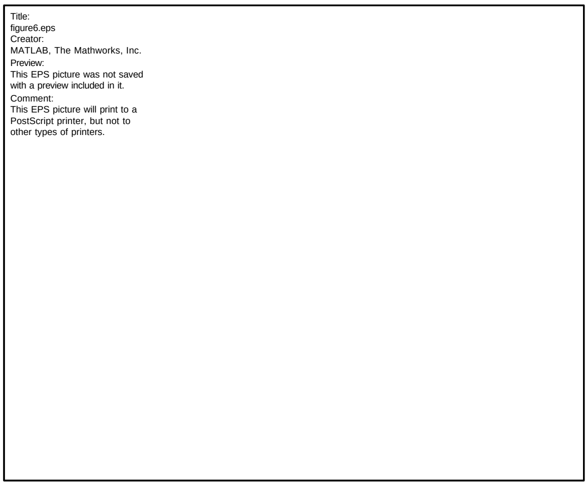

Fig. 7 Steer angle comparison, Barcelona, turn 3.

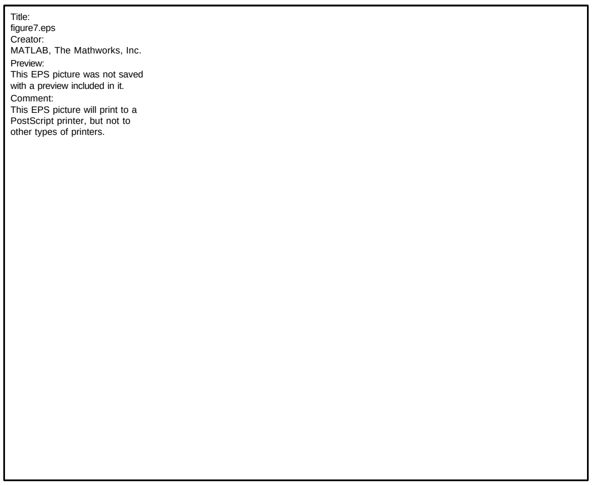

Fig. 8 Yaw rate comparison, Barcelona, turn 3.

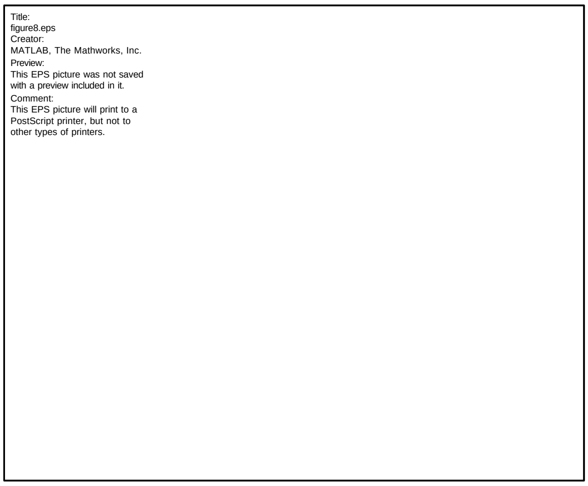

Fig. 9 Lateral velocity comparison, Barcelona, turn 3.

In the final set of figures the different driving strategies computed for a front heavy car and a rear heavy car are compared in some details. Particularly, the focus is on one particular corner, turn number 3 of the Barcelona circuit. This is a long and fast right hand corner, which a Formula One car enters at about 180 [km/h] for the first part and then accelerates all the way out up to speed in excess of 270 [km/s]. Figs 7, 8 and 9 show the comparison of the simulated steer angle control, yaw rate and lateral velocity for the two cars with very different weight distribution as they negotiate the corner. Fig. 10 shows the computed racing lines. The most evident difference is in the steer angle control. As one would expect, the rear heavy car requires far less steering input compared to the front heavy car. The rear heavy car has a cornering attitude which is characterised by a much larger body side slip, as is evident from the lateral velocity, see Fig. 9. The front heavy car goes through the corner with a lower side velocity, which, however, grows rapidly on some occasions.

Although one would expect for the front heavy car greater stability, ensured by the spare force capacity of the rear tyres, in transient condition such spare capacity may not be sufficient and the car may rapidly shift between understeer and oversteer attitude, leading to a less predictable behaviour. The rather noisy yaw rate signal for the front heavy car is also a further indication of such inconsistent behaviour.

5. CONCLUSIONS

A computational suite for determining the minimum possible lap time of a virtual racing car travelling round a defined circuit has been applied to finding the influence of the longitudinal location of the car's mass centre. Variations reported do not include the re-design of suspension stiffnesses, aerodynamic devices etc. in conjunction with the mass centre changes, that, in practice, would be needed to re-define the optimal car. With this limitation, the best mass proportioning for the Barcelona circuit was found to be 39% front, 61% rear, while for Suzuka, it was 38% front, 62% rear. In the neighbourhood of the optimal mass distribution, the sensitivity of the lap time to change is quite low, Figs 2 and 3.

The fastest configuration is distinguishable from the slower ones included, mainly by virtue of its maintaining speed in the faster corners, Fig. 4.

The way in which the mass distribution influences the tyre shear force utilisation at each of the four tyres, on average over the whole circuit, has been highlighted in Figs 5 and 6. The best configuration is characterised by evenness of utilisation over the four tyres. In practice, this would be advantageous also in terms of tyre temperature control and tyre wear.

Steering control inputs required for the relatively rear heavy cars are less and less complex than for the others but the software presumes the perfect driver. Real drivers may have difficulty controlling the yawing motion of the "optimal" car. It may be advisable to set up real cars on the front heavy side of this "optimal", in view of the limitations of real drivers. One possible benefit of the super-driver is that the car can be set up nearer to the truly optimal mass distribution, due to the extra control capability implied.

The computations reflect the controllability problem of the relatively rear heavy cars in requiring more iterative steps to converge to a solution in these cases.

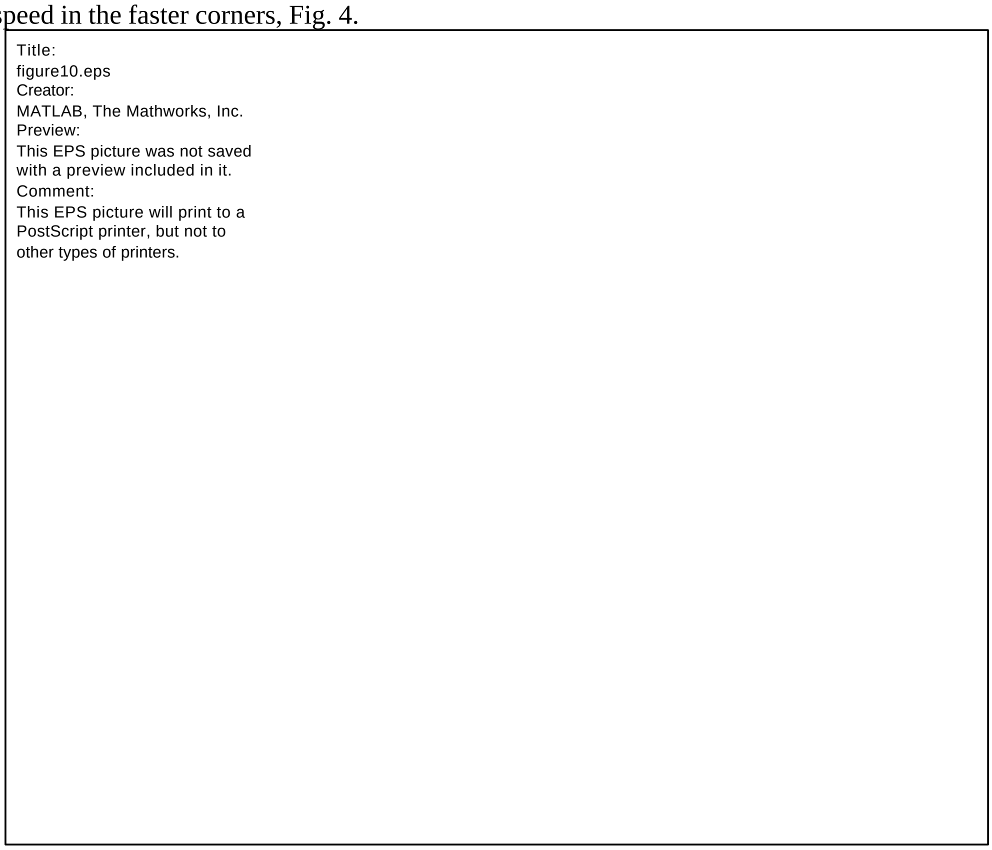

Fig. 10 Racing line comparison on a section of Barcelona circuit.

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