Set-Point Regulation of an Anaerobic Digestion Process with Bounded Output Feedback

R. Antonelli, J. Harmand, J.P. Steyer and A. Astolfi

***Abstract*— This work addresses the robust output feedback tracking problem for a biological wastewater treatment process in the presence of input constraints and limited knowledge on the process parameters. The considered process is an anaerobic upflow fixed bed biological reactor within a semi-industrial scale pilot plant for the treatment of industrial wine distillery wastewater. A mathematical model describing the dynamic behavior of the biological process is considered and nonlinear control techniques are used to design a bounded output feedback control law. Local asymptotic stability and local set point tracking of the resulting closed loop system is proved when the dilution rate is selected as the manipulated variable and the output methane gas flow-rate as the measured and controlled variable. Finally, the implementation of the nonlinear output feedback control law is described and experimental results are reported. It is observed that the proposed controller yields set point regulation, rejects disturbances and preserves stability despite uncertainty on the kinetic parameters.**

***Index Terms*— Continuous Biological Reactors, Wastewater Treatment Plants, Input Constraints, Bounded Output Feedback Control.**

I. INTRODUCTION

Through his industrial and agricultural activities, man has caused many physical, chemical and biological modifications to his environment, some of which have had a harmful effect. Nowadays, stricter environmental rules have been imposed in order to guarantee a limit to the quantity of toxic and organic matter released in industrial and municipal effluents. In the control and improvement of environmental quality, biotechnology is playing a more and more important role through improved or innovative Waste-Water Treatment Plants (WWTP).

Among the available technologies, anaerobic digestion presents a number of relevant advantages. First, this process reduces the Chemical Oxygen Demand (COD) of the influent to produce valuable energy (methane). Secondly, it has been experimentally demonstrated that this process is particularly well adapted for concentrated wastes such as agricultural (*e.g.* plant residues, animal wastes) and food industry wastewater. In addition, it is able to operate under severe conditions, *i.e.* high-strength effluents and short hydraulic retention times. Finally, anaerobic digestion is also often used as a sludge treatment for the stabilization of primary and secondary sludges. This partly explains why there are around 2000 units of “engineered” processes now in operation [1].

It is worth recalling that the first fundamental contributions to the mathematical modeling and control of anaerobic digestion processes have been given by Andrews in [2], [3], whereas further contributions to the solution of the adaptive control and state estimation problems can be found in [4].

Despite these early contributions, the operation of such systems still poses a number of practical problems, see *e.g.* [5], since anaerobic digestion is a complex nonlinear system which is known to be unstable. This has led to quite serious breakdowns when the process was applied in the industry. Furthermore, it must be noted that the actual fundamental drawback in modeling and control of a biological process, and in particular an anaerobic digestion, lies in the difficulty to monitor on-line the key biological variables of the process. If it is relatively simple to measure variables defining the operating conditions such as temperature, pH and flow-rates, it is usually much more difficult to measure component concentrations in the biological reactor. Moreover, during the identification of a real process, one needs to monitor simultaneously both the inputs and the outputs of the process. It is well known that if an advanced sensor is available and capable of measuring a chemical component within the medium, it is easier to implement the sensor at the output of the process rather than at the inputs. This may be justified as follows.

- In most processes, the variability of the measurable parameters is significantly higher at the input than at the output of the process. Although there are some exceptions, it is quite obvious that a given sensor with its intrinsic limited bandwidth will not be able to monitor both the inputs and the outputs with a given calibration data set.
- The presence of Suspended Solids (SS) disturbs most of the advanced available sensors. As a consequence, installation at the output of the reactor is usually more suitable than at the input, even though there is a need for a filtration module in such a case [6].
- When such an advanced instrumentation exists, high investment and operation costs limit drastically the number that can be used. This also explains why most experiments performed to identify a biological process consist of applying a succession of input steps either on the flow-rates or on the input substrate concentrations. In this way only a limited number of off-line measurements are needed (for instance, just before and after each step

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changes) to characterize the input while the available on-line instrumentation is implemented to monitor the output of the process.

Thus, developing control systems only based on simple measurements and able to guarantee performance and stability of the process is of primary importance. The literature on the control of anaerobic digestion can be classified into four main categories (for further information about the different approaches see [7] and references therein).

- Non-specific approaches based on simple measurements. "Non-specific" means that, although innovative, these control systems found their essence in either the field of control theory or in other fields, such as chemical engineering or civil engineering. However, these control laws, except for the linearizing adaptive control in [4], have not been specifically developed for biological systems, since they do not take into account the specific properties of biological systems. This category includes, for example, the application of industrial controllers (such as PID or even on/off) within some simple control strategies such as the regulation of the pH into the reactor. Several linear feedback techniques, such as proportional and proportional-integral control [8], [9], [10] and adaptive control [11], [12], have been proposed. However, these approaches have the disadvantage of neglecting the nonlinearities inherent in most biological processes. Furthermore, they do not provide any guarantee of stability and performance.
- Non-specific approaches based on advanced instrumentation [13]. This category includes, for example, the regulation of alkalinity using simple industrial controllers. These approaches are very interesting but still do not guarantee performance and stability. In addition, their application may be difficult in an industrial environment due to the use of on-line advanced sensors.
- Specific approaches based on the use of advanced sensors. Among them, it is possible to distinguish the interval based approach, which has been successfully applied at semi-industrial scale [14]. Another method has been derived from the theory based on differential geometry [15], [16], [17], [18]. By means of nonlinear coordinate transformations and nonlinear state feedback, the nonlinear biological model may be transformed into an equivalent linear model and linear controller design techniques may then be utilized. Both input-output linearization [19], [20] or state-space linearization [21], [22] have been employed. Several exact linearization techniques have been proposed for the control of fermentation processes. The state-space linearization approach has been applied to single-input/single-output [23] and multi-input/multi-output systems [24]. Adaptive nonlinear controllers based on input-output linearization have also been proposed [25] for the optimization of the biomass production. However, a drawback of most of the nonlinear control techniques proposed to control biological reactors is the need for full knowledge of the kinetics. Obviously, these control strategies are very advanced methods from a control point of view. They use both advanced control methodologies and powerful advanced instrumentation. If they are very interesting from an academic point of view, there is no doubt that their industrial implementation is yet to come.
- Specific approaches based on simple industrial measurements. This category includes methods that allow the designer to guarantee performance and stability of the process in the presence of uncertainty and disturbance by using only simple industrial measurements. The approach presented in this work belongs to this last class.

Component	Range
Volatile Fatty Acids (g/l)	$[1.50 - 10.0]$
Total COD (g/l)	$[9.00 - 34.0]$
Soluble COD (g/l)	$[7.60 - 31.0]$
Total Suspended Solids (g/l)	$[2.40 - 10.0]$
Volatile Suspended Solids (g/l)	$[1.20 - 5.40]$
pH	$[4.50 - 5.40]$

TABLE I
COMPOSITION OF THE INDUSTRIAL WINE DISTILLERY WASTEWATER TREATED IN THE PILOT PLANT.

The proposed method, which achieves the stabilization of continuous biological reactors independently of the process kinetics, is an output feedback control approach. In this case, process stabilization can be achieved by regulating the concentration of the biological variables available for measurement, *i.e.* substrate or biomass, by using the dilution rate as the control action [26], [25], [27].

The paper is organized as follows. The description of the pilot plant and the specification of the operating conditions are given in Section II. Section III describes the mathematical model used to represent the dynamics of the biological wastewater treatment process. This is followed, in Section IV, by the description of an output feedback control design method yielding local set point regulation and bounded control action. Finally, experimental results are discussed in Section V and Section VI contains some conclusions.

II. PROCESS DESCRIPTION

The pilot plant located at the *Laboratoire de Biotechnologie de l'Environnement LBE–INRA* in Narbonne is a continuous up-flow fixed bed reactor (Figure 1). This reactor passes fluid through a bed of micro-organisms held on a solid support, hence the name *fixed bed* reactor. The experiments have been carried out by treating raw industrial wine distillery effluent obtained from wineries in the area of Narbonne. This substrate has been chosen because of its availability in the Narbonne area. Its main characteristics during all the process are summarized in Table I.

As described in [28], [29], [6], the continuous fixed bed reactor is a circular column 3.5 m high, 0.6 m in diameter and with a volume of 0.982 m^3. The support utilized in the fixed bed is *Cloisonyl* with a specific surface of 180 m^2/m^3, filling 0.0337 m^3 of the reactor volume and leaving 0.9483 m^3

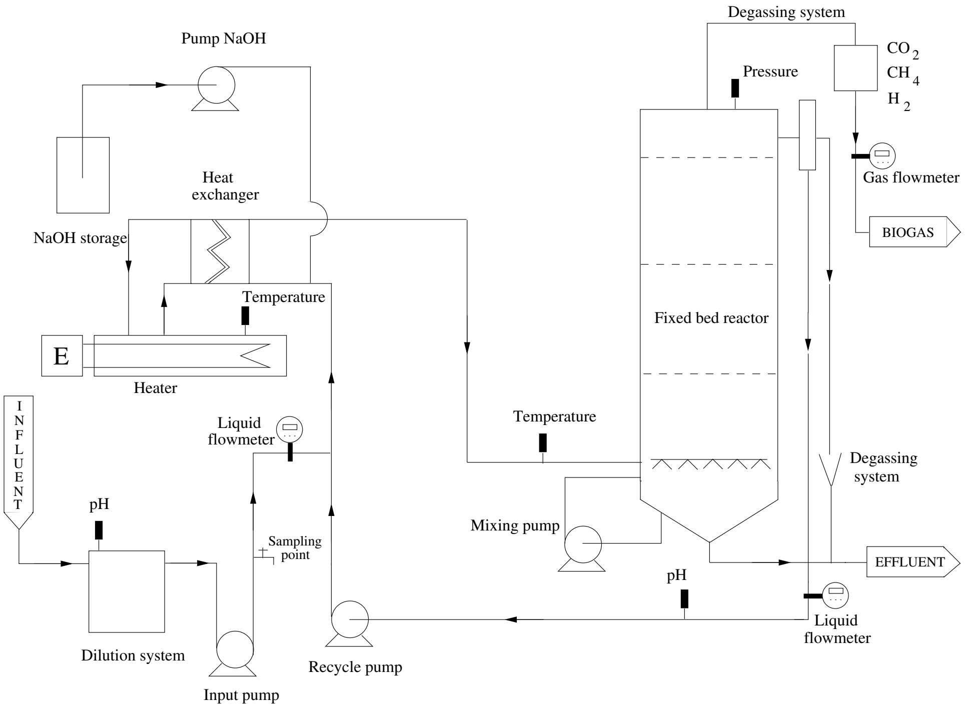

Fig. 1. Schematic of the biological wastewater treatment pilot plant in Narbonne, France.

of effective volume. This support is a multichannel tubular polyvinylchloride (PVC) structure. The pilot plant also includes a reactor feeding system with three tanks of 27 m^3 each, situated outside the laboratory and connected to the plant by a piping system 0.1 m in diameter and 60 m long.

The dilution system, installed at the entrance of the reactor, is depicted in detail in Figure 2. This system, built at the *LBE–INRA*, consists of a 0.2 m^3 dilution tank with a float in order to control the level; two pneumatic valves which manipulate the water and raw wastewater flow-rates; and some level sensors directly connected to the control system.

Connected to this tank is a remotely controllable peristaltic pump that ensures the desired influent flow-rate is fed into the reactor and allows to disturb the process in a controlled manner.

Fresh substrate is mixed with the recycled stream and then the mixture is sent to a heat exchanger which regulates the stream temperature at the optimal temperature of 35 °C using a local PID controller. The heated liquid is then introduced at the bottom of the reactor where it is homogenized by the mixing pump. Inside the reactor, the liquid passes through the Cloisonyl structure until it reaches the top. Finally, the liquid from the top of the reactor is collected by overflow into a receiving vessel. At this stage, a degassing system, composed of a three-phase separator, separates the gas from the liquid and decants the suspended solids. Some of the liquid is sent to the sewer, while the rest is recycled at the constant flow-rate of about 150 l/h.

The system used to measure and control the *pH* in the recycling stream includes a *pH* sensor (*HANNA Instruments*), a Proportional-Integral-Derivative (PID) controller, a caustic soda (NaOH) storage tank and a dosing pump. The addition of soda at 50% may take place in the dilution system or in the recirculation loop just before the heat exchanger.

Input and recycling liquid flow-rates are measured by two electro-magnetic sensors (*KRHONE*) with normalized analog outputs $(4 - 20$ $mA)$.

The gas loop (Figure 3) is composed of several units. A dryer removes humidity by cooling down the gas. A gas flow-meter, located at the output of the analysis loop, employs an electro-magnetic float to continuously measure the gas produced. An automatic H_2 analyzer is also connected but is not described here since it is not used by the control system. An ULTRAMAT 22 P sensor (*SIEMENS*) measures the percentage composition of carbon dioxide, CO_2, and methane, CH_4, in the gas. This sensor works on the principle of non-dispersive absorption of infrared light. Finally, a peristaltic pump is used to ensure a fixed gas flow-rate of 8 l/h throughout the electro-chemical cell.

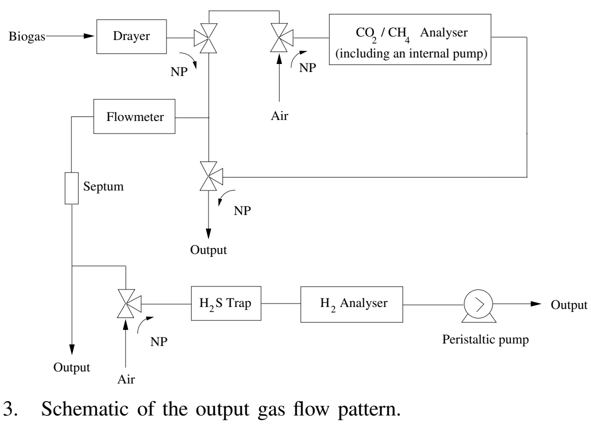

Fig. 3. Schematic of the output gas flow pattern.

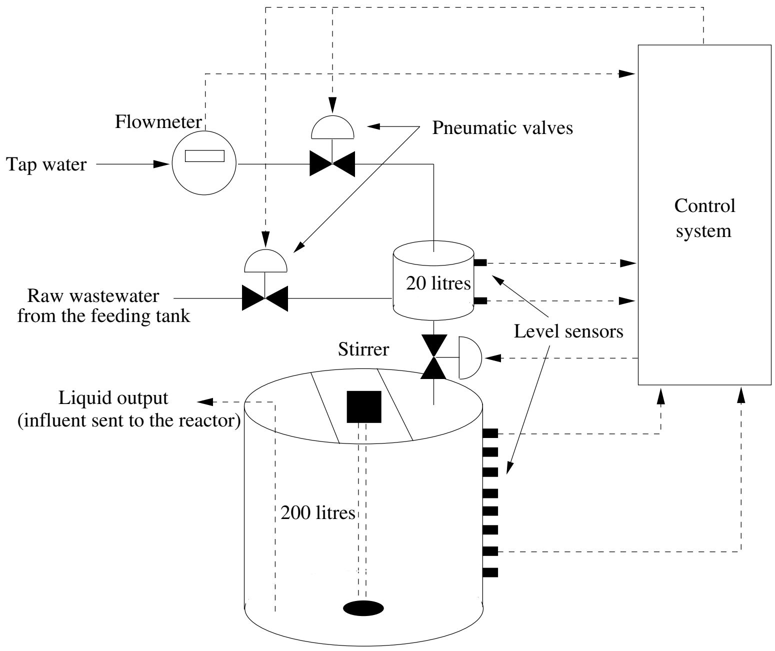

Fig. 2. Schematic representation of the dilution system.

Several on-line measurements are available:

- input liquid flow-rate;
- recycled liquid temperature;
- heater temperature;
- *pH* of the recycled liquid;
- output gas flow-rate;
- biogas composition (CO_2/CH_4 percentage and H_2 concentration).

All these sensors and actuators are connected to an input/output device that allows the acquisition, treatment and storage of data on a PC using the *Modular SPC*® software, developed by the *LBE–INRA*. This software may perform advanced control law calculations as well as process supervision. It provides an operator interface, manages the regulatory control loops and performs the data logging. The human operator can introduce and change some process parameters such as the sampling time, the number of required measurements and the time after which the control action is effective.

III. Problem Formulation

In general the goal of a wastewater biological treatment is to employ micro-organisms to decompose, usually by oxidation, the organic compounds contained inside the effluent and therefore to reduce the pollutant concentration in the outlet stream below a specified value, usually fixed by environmental and safety regulations.

The level of decomposable organic matter in industrial or municipal wastewater is measured in terms of its *Biological Oxygen Demand* (BOD) or *Chemical Oxygen Demand* (COD). The BOD measures the milligrams of oxygen consumed during the biological decomposition of organic matter occurring in a litre of wastewater during a specified period of time, usually five days. The COD determines the milligrams of oxygen per litre of wastewater which are consumed during the oxidation of the organic matter by hot acidified dichromate.

In particular, the COD is the key variable in the anaerobic digestion process for the treatment of industrial wine distillery wastewater described here. In the first analysis, the global process may be described by the following mathematical model [30]:

$$\left\{\begin{array}{rcl} \dot{X}_1 & = & [\mu_1(S_1) - \alpha D]\, X_1 \\ \dot{X}_2 & = & [\mu_2(S_2) - \alpha D]\, X_2 \\ \dot{S}_1 & = & D(S_1^{in} - S_1) - k_1\mu_1(S_1)X_1 \\ \dot{S}_2 & = & D(S_2^{in} - S_2) + k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2 \end{array}\right. \quad (1)$$

where $X_1(g/l)$ is the concentration of acidogenic bacteria, $X_2(g/l)$ is the concentration of methanogenic bacteria, $S_1(g/l)$ is the concentration of COD, $S_2(g/l)$ is the concentration of Volatile Fatty Acids (VFA), $S_1^{in}(g/l)$ is the input concentration of COD, $S_2^{in}(mmol/l)$ is the input concentration of VFA, $0 < \alpha \leq 1$ is a proportionality parameter of experimental determination, $D(day^{-1})$ is the dilution rate, $k_1(gCOD/gX_1)$ is the yield coefficient for COD degradation, $k_2(mmolVFA/gX_1)$ is the yield coefficient for fatty acid production, $k_3(mmolVFA/gX_2)$ is the yield coefficient for fatty acid consumption.

The nonlinear behavior is given by the two specific microbial growth rates, $\mu_1(S_1)$ and $\mu_2(S_2)$, expressed by the Monod law and the Haldane law respectively, *i.e.*

$$\mu_1(S_1) = \frac{\mu_{m1}S_1}{K_{S1} + S_1} \qquad \mu_2(S_2) = \frac{\mu_{m2}S_2}{K_{S2} + S_2 + \left(\dfrac{S_2}{K_{I2}}\right)^2} \quad (2)$$

where $\mu_{m1}(day^{-1})$ is the maximum acidogenic biomass growth rate, $\mu_{m2}(day^{-1})$ is the maximum methanogenic

biomass growth rate, $K_{S1}(CODg/l)$ is the saturation parameter associated with S_1, $K_{S2}(mmolVFA/l)$ is the saturation parameter associated with S_2, and $K_{I2}(mmolVFA/l)$ is the inhibition parameter associated with S_2.
A measurement available on-line in this model is the *output gas flow-rate*,

$$Q_{GAS} = q_C + q_M \tag{3}$$

given by the sum of the carbon dioxide flow-rate, q_C, and of the methane flow-rate, q_M, produced inside the biological reactor. In particular, the methane gas flow-rate

$$q_M = q_M(X_2, S_2) = k_6\mu_2(S_2)X_2 \tag{4}$$

will be the measured variable considered to implement the output feedback controller designed in Section IV-A. The goal of the controller is to regulate the variable q_M to a desired set point, which is in general specified by environmental regulations.

IV. Control design

This section describes the extension of the nonlinear control design technique proposed in [31], [26], to the anaerobic digestion process (1). In this process, modeled by four nonlinear ordinary differential equations derived from mass balance considerations, the dilution rate D is the manipulated variable. This variable appears in all the differential equations of the model and the presence of the proportionality parameter α makes the stability analysis of the process quite complex. For simplicity, the control design problem for system (1) will be studied for two different cases: when the proportionality parameter α is assumed unitary and when $0 < \alpha < 1$.

A. The case $\alpha = 1$

Consider the system (1) with $\alpha = 1$, *i.e.*

$$\begin{cases} \dot{X}_1 &=& [\mu_1(S_1) - D]\, X_1 \\ \dot{X}_2 &=& [\mu_2(S_2) - D]\, X_2 \\ \dot{S}_1 &=& D(S_1^{in} - S_1) - k_1\mu_1(S_1)X_1 \\ \dot{S}_2 &=& D(S_2^{in} - S_2) + k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2 \end{cases} \tag{5}$$

These equations describe a class of Continuous Stirred Tank Reactors (CSTR) widely used and studied in biotechnology. Let

$$\begin{array}{rcl} Z_1 &=& S_1^{in} - S_1 - k_1X_1 \\ Z_2 &=& S_2^{in} - S_2 - k_3X_2 + k_2X_1 \end{array} \tag{6}$$

be two *auxiliary* variables and note that, with the use of these new variables, system (5) can be described by the equations

$$\begin{cases} \dot{Z}_1 &=& -DZ_1 \\ \dot{Z}_2 &=& -DZ_2 \\ \dot{X}_1 &=& \left[\mu_1\left(S_1^{in} - Z_1 - k_1X_1\right) - D\right] X_1 \\ \dot{X}_2 &=& \left[\mu_2\left(S_2^{in} - Z_2 - k_3X_2 + k_2X_1\right) - D\right] X_2 \end{cases} \tag{7}$$

The main assumptions made in this work can be summarized in the following way.

A1) For any control input $D \geq 0$, the state variables X_1, X_2, S_1 and S_2 are non-negative, *i.e.*

$$X_1 \geq 0, \quad X_2 \geq 0, \quad S_1 \geq 0, \quad S_2 \geq 0.$$

As a consequence the auxiliary variables Z_1 and Z_2 are such that

$$Z_1 \leq S_1^{in}, \qquad Z_2 \leq S_2^{in} + k_2\overline{X}_1$$

where $\overline{X}_1$ is the maximum value of the state variable X_1.

A2) The substrate concentration set points, S_1^{sp} and S_2^{sp}, are such that the corresponding dilution rate set point D^{sp} satisfies simultaneously the two relations

$$\begin{array}{rcl} D^{sp} &<& \max\limits_{S_1} \mu_1(S_1) \\ D^{sp} &<& \max\limits_{S_2} \mu_2(S_2). \end{array}$$

A3) The control input D takes values inside a non-empty closed subset of the positive real axis, *i.e.*

$$D \in [\underline{D}, \overline{D}],$$

for some known $\underline{D}$ and $\overline{D}$ such that $0 < \underline{D} < \overline{D}$. Moreover $D^{sp} \in [\underline{D}, \overline{D}]$.

A4) The yield coefficients k_1, k_2 and k_3 are constant and the feed substrate concentrations S_1^{in} and S_2^{in} are piecewise constant.

In this case, it is convenient to analyze the stability of the system (5) by considering the sub-system in the (Z_1, X_1) variables [26], *i.e.*

$$\begin{cases} \dot{Z}_1 &=& -DZ_1 \\ \dot{X}_1 &=& \Big[\mu_1\left(S_1^{in} - Z_1 - k_1X_1\right) - D\Big] X_1 \end{cases} \tag{8}$$

and the sub-system

$$\begin{cases} \dot{Z}_2 &=& -DZ_2 \\ \dot{X}_2 &=& \Big[\mu_2\left(S_2^{in} + k_2X_1 - Z_2 - k_3X_2\right) - D\Big] X_2. \end{cases} \tag{9}$$

Note that, by Assumption **A3)**, Z_1 converges exponentially to zero. Moreover, $X_1(t)$ is such that, for any $\epsilon > 0$ there is $t_\epsilon \geq 0$ such that

$$X_1(t) \leq \frac{S_1^{in} + \epsilon}{k_1}$$

for any $t \geq t_\epsilon$. As a result, the quantity k_2X_1, appearing in the argument of the specific microbial growth rate μ_2, may be interpreted as a *disturbance* on the feed substrate concentration S_2^{in}. Finally, the output variable (4) may be rewritten in terms of the auxiliary variable Z_2 as

$$q_M = q_M(X_1, X_2, Z_2) = k_6\mu_2\left(S_2^{in} + k_2X_1 - Z_2 - k_3X_2\right)X_2.$$

Proposition 1: Consider the system described by equations (9). Suppose that Assumptions **A1)**, **A2)**, **A3)** and **A4)** hold. Assume moreover that q_M is the measured variable and q_M^{sp} is its desired set point value. Then, for any $\theta(0) \in (\underline{D}, \overline{D})$, the dynamic output feedback control law

$$\begin{cases} D &=& \theta \\ \dot{\theta} &=& k_{q_M}(q_M - q_M^{sp})(\theta - \overline{D})(\theta - \underline{D}) \end{cases} \tag{10}$$

with $k_{q_M} > 0$, is such that the closed loop system (9)-(10) is *locally stable* around the equilibrium point $(Z_2^{sp}, X_2^{sp}, \theta^{sp})$. Moreover, for any initial condition $(Z_2(0), X_2(0))$ in the region

$$\sigma'' = \left\{(Z_2, X_2) \in I\!R^2 \mid Z_2 + k_3 X_2 < S_2^{in} + k_2\overline{X}_1, \quad 0 < X_2 < \frac{(S_2^{in} + k_2\overline{X}_1)}{k_3}\right\}, \tag{11}$$

$(Z_2(t), X_2(t))$ remains in this region and, for any $X_2(0)$ sufficiently close to X_2^{sp} the trajectories of the closed loop system (9)-(10) converge asymptotically to (Z_2^{sp}, X_2^{sp}) and q_M converges to q_M^{sp}. Finally, the control variable D remains in the set $[\underline{D}, \overline{D}]$ for all $t \geq 0$.

***Remark** 1:* The control law in equation (10) is a nonlinear PI controller. To show this fact note that

$$u = \frac{\overline{u}\,[u(0) - \underline{u}] - \underline{u}\,[u(0) - \overline{u}]\,\phi_{nl}(y)}{u(0) - \underline{u} - [u(0) - \overline{u}]\,\phi_{nl}(y)}$$

with

$$y = q_M - q_M^{sp},$$

$$\phi_{nl}(y) = \exp\left[k(\overline{u} - \underline{u})\int_0^{t_f} y(t)dt\right]$$

and $u(0)$ is the value of the control input at the initial time. Note moreover that the role of the term

$$(\theta - \overline{D})(\theta - \underline{D})$$

is to keep $\theta(t)$ in the set $(\underline{D}, \overline{D})$ for any t, *i.e.* it implements a *continuous* projection of the state θ in the set $(\underline{D}, \overline{D})$.

***Remark** 2:* The control law (10) does not require the knowledge of any system parameter, *i.e.* the claims in Proposition 1 hold for any value of the parameters k_i and of the parameters in the functions $\mu_i(\cdot)$. However, the size of the set σ'' and the set of *good* initial conditions, *i.e.* of initial conditions for which asymptotic regulation is achieved, depends upon such parameters.

Proof. Observe that the Z_2 sub-system of (9) does not depend upon S_2 and that, by Assumption **A3)**, this sub-system is globally asymptotically stable. Hence, consider the system composed of the X_2 sub-system of the system (9) with $Z_2 = 0$, and of the control law (10), *i.e.*

$$\left\{\begin{array}{rcl} \dot{X}_2 &=& \left[\mu_2\left(S_2^{in} + k_2X_1 - k_3X_2\right) - \theta\right] X_2 \\ \dot{\theta} &=& k_{q_M}(q_M - q_M^{sp})(\theta - \overline{D})(\theta - \underline{D}) \end{array}\right. \tag{12}$$

Choosing the candidate Lyapunov function

$$W = -\int_{X_2^{sp}}^{X_2}\left[\frac{q_M(\zeta_2) - q_M^{sp}}{\zeta_2}\right]d\zeta_2 + \omega(\theta) \tag{13}$$

with $q_M(\zeta_2) = k_6\mu_2(S_2^{in} + k_2X_1^{sp} - k_3\zeta_2)\zeta_2$, $q_M^{sp} = k_6\mu_2(S_2^{in} + k_2X_1^{sp} - k_3X_2^{sp})X_2^{sp}$,

$$\omega(\theta) = \ln(\overline{D} - \theta)^{\tau/k_{q_M}} - \ln(\theta - \underline{D})^{(\tau+1)/k_{q_M}}$$

and[1]

$$\tau = \frac{D^{sp} - \underline{D}}{\overline{D} - \underline{D}} < 0,$$

one gets

$$\begin{aligned} \dot{W} &= -(q_M(X_2) - q_M^{sp})\left[\mu_2\left(S_2^{in} + k_2X_1 - k_3X_2\right) - D\right] + \\ &\quad (D^{sp} - D)\left(q_M(X_2) - q_M^{sp}\right) \\ &= -\left(q_M(X_2) - q_M^{sp}\right)\left[\mu_2\left(S_2^{in} + k_2X_1 - k_3X_2\right) - D^{sp}\right] \\ &= -\frac{1}{k_6}\left(q_M(X_2) - q_M^{sp}\right)\left(\frac{q_M(X_2)}{X_2} - \frac{q_M^{sp}}{X_2^{sp}}\right). \end{aligned} \tag{14}$$

Set

$$q_M = q_M^{sp} + \delta q_M \qquad X_2 = X_2^{sp} + \delta X_2 \tag{15}$$

and note that, for small δX_2 and δq_M, one has $\delta X_2 = -\lambda\delta q_M$ for some $\lambda > 0$. Then

$$\dot{W} = -\frac{1}{k_6}\delta q_M\left(\frac{q_M^{sp} + \delta q_M}{X_2^{sp} + \delta X_2} - \frac{q_M^{sp}}{X_2^{sp}}\right)$$

and, by

$$\frac{1}{X_2^{sp} + \delta X_2} \simeq \frac{1}{X_2^{sp}} - \frac{1}{(X_2^{sp})^2}\delta X_2$$

one has

$$\dot{W} = -\frac{1}{k_6}(\delta q_M)^2\left[\frac{1}{X_2^{sp}} + \frac{\lambda q_M^{sp}}{(X_2^{sp})^2} + \frac{\lambda\delta q_M}{(X_2^{sp})^2}\right] + h.o.t.(\delta q_M)$$

and the term in square bracket is positive. Hence, the closed loop system (9)-(10), which is a cascaded interconnection of two locally stable systems, is also locally stable. Finally, from the previous discussion and the structure of the dynamic control law, it is easy to conclude that the set σ'' is positively invariant and that for every initial condition $(Z_2(0), X_2(0)) \in \sigma''$, with $X_2(0)$ sufficiently close to X_2^{sp}, and $\theta(0) \in (\underline{D}, \overline{D})$

$$\lim_{t\to\infty} Z_2(t) = 0 \quad \lim_{t\to\infty} q_M(t) = q_M^{sp} \quad \lim_{t\to\infty} \theta(t) = \theta_\infty$$

with $\theta_\infty \in [\underline{D}, \overline{D}]$. $\lhd$

***Remark** 3:* The Lyapunov function (13) is inspired by the general adaptive control result developed in [32].

B. The case $0 < \alpha < 1$

The stability analysis becomes more complex when the proportionality parameter $\alpha \neq 1$. Consider the sub-system in the (X_2, S_2) variables, *i.e.*

$$\left\{\begin{array}{rcl} \dot{X}_2 &=& \Big[\mu_2(S_2) - \alpha D\Big] X_2 \\ \dot{S}_2 &=& D(S_2^{in} - S_2) + k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2. \end{array}\right. \tag{16}$$

It is straightforward to show that the term $k_2\mu_1(S_1)X_1$ is bounded and may be seen as a limited disturbance upon the substrate concentration S_2. Hence, it is reasonable to assume that the bounded term $k_2\mu_1(S_1)X_1$ has a negligible effect

[1]Note that the function $\omega(\theta)$ is positive for any $\theta \in (\underline{D}, \overline{D})$, $\lim_{\theta\to\underline{D}^+} \omega(\theta) = +\infty$ and $\lim_{\theta\to\overline{D}^-} \omega(\theta) = +\infty$.

on the dynamic behavior of the variable S_2. Hence the term $k_2\mu_1(S_1)X_1$ will be neglected in the following discussion.

***Proposition** 2:* Consider the closed loop system

$$\begin{cases} \dot{X}_2 &= \Big[\mu_2(S_2) - \alpha\theta\Big] X_2 \\ \dot{S}_2 &= \theta(S_2^{in} - S_2) - k_3\mu_2(S_2)X_2 \\ \dot{\theta} &= k_{q_M} k_6(\theta - \overline{D})(\theta - \underline{D}) \left[\mu_2(S_2)X_2 - \mu_2(S_2^{sp})X_2^{sp}\right] \end{cases} \tag{17}$$

with

$$\mu_2(S_2) = \frac{\mu_0 S_2}{K_S + S_2 + \dfrac{S_2^2}{K_I}} \tag{18}$$

and assume that

$$S_2^{in} > S_2^{sp} + \frac{\mu_2^{sp}}{\left.\dfrac{\partial \mu_2}{\partial S_2}\right|_{sp}} \tag{19}$$

with $\mu_2^{sp} = \mu_2(S_2^{sp})$. Moreover, suppose that Assumptions **A1)**, **A2)**, **A3)** and **A4)** hold and that q_M is the measured variable. Then the system (17) is *locally stable* around the equilibrium point $(X_2^{sp}, S_2^{sp}, \theta^{sp})$ and every trajectory starting sufficiently close to $(X_2^{sp}, S_2^{sp}, \theta^{sp})$ is such that

$$\lim_{t\to\infty} X_2(t) = X_2^{sp} \qquad \lim_{t\to\infty} q_M(t) = q_M^{sp} \qquad \lim_{t\to\infty} \theta(t) = \theta_\infty$$

where $\theta_\infty \in [\underline{D}, \overline{D}]$.

Proof. The closed loop system linearized around the equilibrium point

$$(X_2^{sp}, S_2^{sp}, \theta^{sp}) = \left(\frac{S_2^{in} - S_2^{sp}}{\alpha k_3}, S_2^{sp}, \frac{\mu_2^{sp}}{\alpha}\right)$$

has the characteristic polynomial

$$\lambda^3 + \lambda^2\left(\frac{\mu_2^{sp} + ab}{\alpha}\right) +$$

$$\lambda\left[\frac{ab\mu_2^{sp}}{\alpha} + \frac{k_\theta k_6 ac}{k_3}\left(\mu_2^{sp} - \frac{ab}{\alpha}\right)\right] + \frac{k_\theta k_6 ac\mu_2^{sp}}{\alpha k_3}\left(\mu_2^{sp} - ab\right)$$

with

$$a = (S_2^{in} - S_2^{sp}) > 0, \qquad b = \left.\frac{\partial \mu_2}{\partial S_2}\right|_{sp} > 0$$

and

$$c = \left(\frac{\mu_2^{sp}}{\alpha} - \overline{D}\right)\left(\frac{\mu_2^{sp}}{\alpha} - \underline{D}\right) < 0.$$

The claim is therefore proved using Routh test. $\triangleleft$

It must be noted that the control law (10) has only three design parameters, namely $\overline{D}$, $\underline{D}$ and k_{q_M}. Moreover, in general, $\overline{D}$ and $\underline{D}$ are determined by the specific application, *i.e.* are fixed by the limits of the actuator, hence the only parameter available for tuning is k_{q_M}. This implies, on one hand, that the control law (10) is very simple to tune, and that the influence of the parameter k_{q_M} on the closed loop system performance can be easily evaluated. On the other hand, this limited flexibility may yield performance lower than that attainable by other more complicated designs. Nevertheless, as shown in the next section, the proposed controller yields good set point tracking properties and it is robust against the presence of unmodeled dynamics and sensor noise. Finally, it is worth mentioning that the tuning process of the controller requires only a small amount of time, and the parameter k_{q_M} can also be changed during the process operation without compromising closed loop stability.

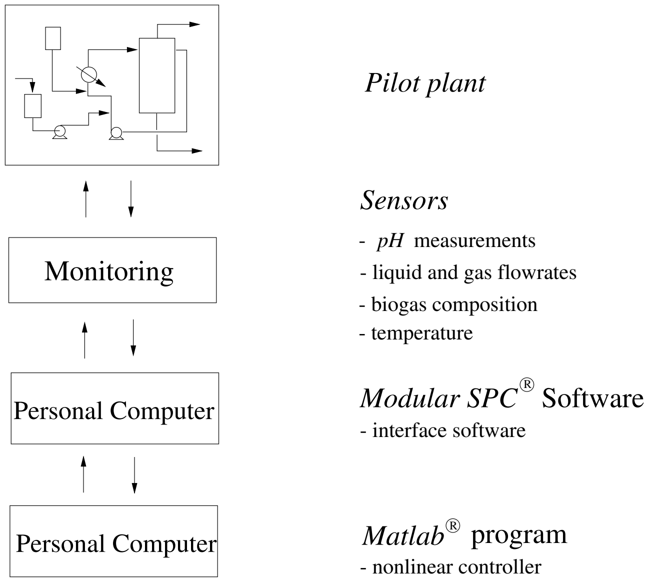

Fig. 4. Experimental implementation of the nonlinear controller (20).

V. Experimental Details

This section details the implementation of the nonlinear controller (10), *i.e.*

$$\begin{cases} D = & \theta \\ \dot{\theta} = & k_{q_M}(q_M - q_M^{ss})(\theta - \overline{D})(\theta - \underline{D}) \end{cases} \tag{20}$$

on the anaerobic digestion process, which is carried out in the continuous fixed bed reactor of the pilot plant and modeled by the system (1). A schematic diagram of the implementation procedure is given in Figure 4 (see also [33] for further detail and programs listing.)

The results of the experiments are illustrated in Figures 5 and 6. Figure 5 shows the behavior of the measured variable q_M, which is the methane flow-rate produced in the process, when the control law (20) is active, whereas Figure 6 depicts the time history of the manipulated variable $F_{in} = DV$, where F_{in}, D and V are the fresh feed flow-rate, the dilution rate and the volume of the fixed bed reactor, respectively. The sampling period is fixed at two minutes and the experiment consists of four changes in the output gas flow-rate set point, q_M^{ss}:

- from open loop condition to 65 l/h,
- from 65 l/h to 80 l/h,
- from 80 l/h to 120 l/h,
- from 120 l/h to 65 l/h.

For simplicity and to enable a better description of all the events, the experiment has been divided into the four intervals, ***I1***, ***I2***, ***I3*** and ***I4*** (see Figures 7, 8 and 9).

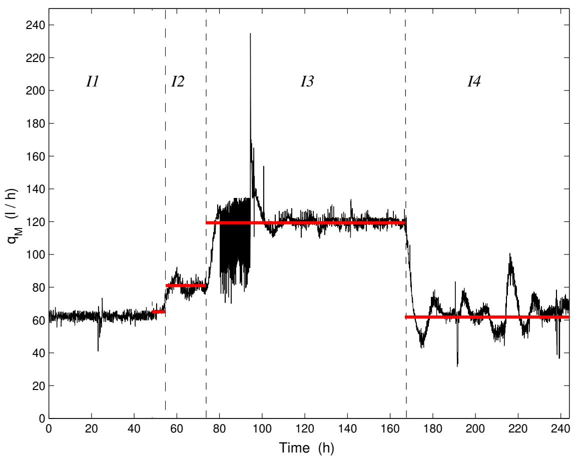

Fig. 5. Experimental data for the output gas flow-rate q_M. Set point: bold line. Note that from $t = 0$ h to $t = 53$ h the process is in open loop, *i.e.* no set point is specified.

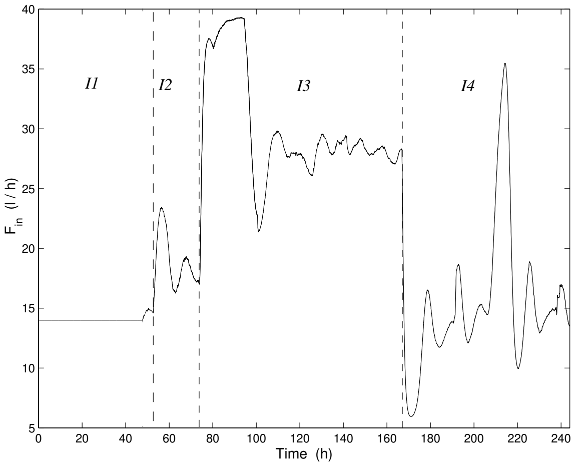

Fig. 6. Experimental data for the manipulated variable F_{in}.

- ***I1*** ($0\ h \leq t < 53\ h$) From the initial time $t = 0\ h$ until $t = 48\ h$ (see Figure 7) the process is in open loop with $F_{in} = 14\ l/h$.
  At $t = 48\ h$ the set point of the output gas flow-rate is set to $q_M^{ss} = 65\ l/h$.
- ***I2*** ($53\ h \leq t < 74\ h$) At $t = 53\ h$ the set point of the output gas flow-rate is changed to $q_M^{ss} = 80\ l/h$.
- ***I3*** ($74\ h \leq t < 167\ h$) Another variation of set point is at $t = 74\ h$ (see Figure 8) when q_M^{ss} changes from $80\ l/h$ to $120\ l/h$.
  From $t = 80\ h$ until $t = 94\ h$ the output gas sensor does not work properly, yielding large oscillations in the gas flow-rate measurements. This is due to the high pressure at the top of the reactor caused by the presence of foam. Each peak is the result of the automatic opening of the valve at the top to reduce the pressure. The gas is removed without passing through the gas analysis loop. This implies a change in the output gas path which affects the measurements of the output gas flow-rate.
  At $t = 94\ h$ the problem is solved by adding an anti-foaming agent and by opening the gas analysis loop. This can be interpreted as a strong disturbance which causes a rapid decrease of F_{in}. In fact, with the set point of the output gas flow-rate still at $120\ l/h$, the control input F_{in} changes from almost $40\ l/h$ to $30\ l/h$. The effect of this disturbance is easily observable by examining the profile of the output gas flow-rate, q_M, between $t = 94\ h$ and $t = 109\ h$. After $t = 109\ h$, q_M again reaches the set point value of $120\ l/h$.
- ***I4*** ($167\ h \leq t \leq 244\ h$) At $t = 167\ h$ (see Figure 9) there is another change in the set point q_M^{ss}, which is decreased from $120\ l/h$ to $65\ l/h$. Despite the large excursion of the set point, the system remains stable and the new set point is attained.
  Just before $t = 190\ h$ there is a disturbance which causes a perturbation in the value of the output gas flow-rate. This is due to the fact that the gas analysis loop was manually opened to remove the water accumulated by condensation in the pipes. In this case, the controller also rejects the disturbance.
  At $t = 206\ h$ there is a problem with the dilution system: only water is added to the process rather than a mixture of water and vinasses. The concentration in the feeding stream decreases and, to reject this disturbance, the control system increases the input variable F_{in}.
  At $t = 214\ h$ the dilution tank is emptied and new wastewater is fed to the process, this time without additional water. Since the feed concentration is now back to normal, the output gas flow-rate increases and, consequently, the input F_{in} is decreased by the controller. Also the *pH* decreases because of a small over-feeding of the process while F_{in} decreases to its set point value.
  Finally, at $t = 238\ h$ a small disturbance on the output gas flow-rate measurement is effectively rejected by the control system by increasing the input variable F_{in}.
  The experiment terminates at $t = 244\ h$.

The nonlinear controller has been implemented in a discrete-time form within the *Modular SPC*® software using a *Matlab*® program.

VI. Conclusions

This work has addressed the output feedback regulation of an anaerobic up-flow fixed bed biological reactor for the treatment of industrial wine distillery wastewater. The pilot plant, located at the *Laboratoire de Biotecnologie de l'Environnement de l'INRA* in Narbonne, France, has been described in detail together with the operating conditions. Its mathematical model has been studied and a nonlinear output feedback control law derived. The proposed controller feeds back only the available measurement which is the output methane gas flowrate and yields bounded control action. No exact knowledge of the kinetic model parameters has been required and local asymptotic stability and local set point tracking for the resulting closed loop system have been proved. Experimental results obtained by implementing the nonlinear output feedback controller on the anaerobic digestion process have also been discussed in detail. The results demonstrate that the nonlinear controller is able to reject unmeasured disturbances and noisy measurements, provides a consistent closed loop response to step

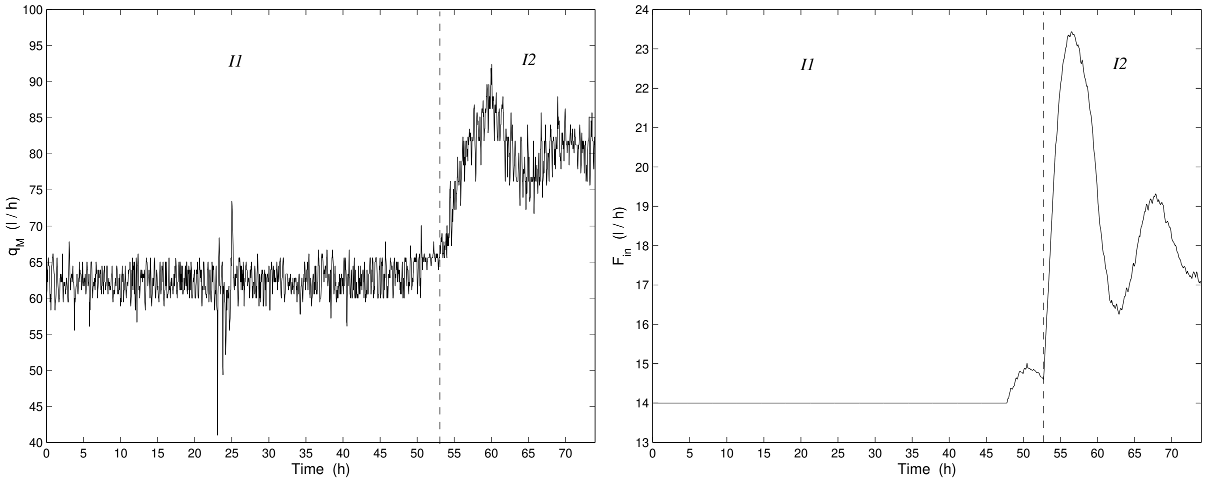

Fig. 7. Intervals ***I1*** and ***I2***: output gas flow-rate q_M and control input F_{in}.

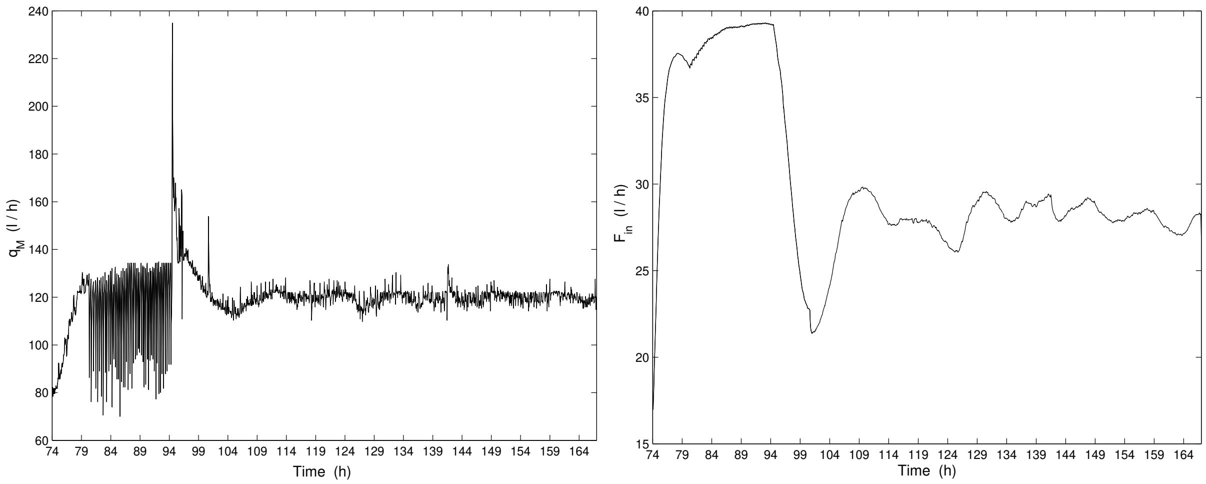

Fig. 8. Interval ***I3***: output gas flow-rate q_M and control input F_{in}.

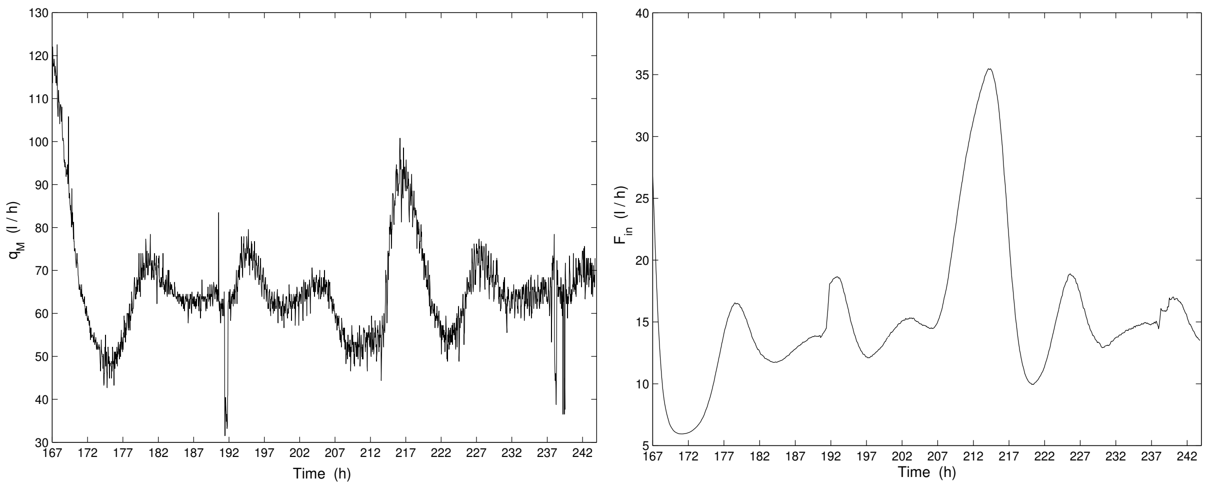

Fig. 9. Interval ***I4***: output gas flow-rate q_M and control input F_{in}.

changes in the set point and preserve (local) stability despite model mismatches between the mathematical model used for designing the controller and the plant. We also point out that the controller has a simple structure and can be easily tuned during the process operation.

These last properties, together with the guaranteed stability properties discussed in Section IV, render the proposed controller a natural candidate whenever it is necessary to implement and tune a controller in a limited amount of time or whenever a fault, hindering the applicability of more complex control methods, occurs in the plant.

The problem of the design of an output feedback control law yielding global regulation, despite input constraints and unknown parameters, is still open, and it is currently under investigation.

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