Two-level Control of Processes with Dead Time and Input Constraints*

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Abstract

This paper proposes a new idea, *shaping the control signal*, and generalizes the time-delay-filter-based deadbeat control for processes with dead time to *two-level control* so that the controller saturation is avoided. The controller mimics experienced manual operations to provide a two-level control signal. At first, the controller outputs a value close to the maximal value and then the controller outputs a smaller value to maintain the steady-state output at the set point. The system output reaches the steady state in finite time, which is explicitly determined by the upper bound of the control output and is independent of the controller and (almost) of the sampling period. The disturbance response can be freely tuned according to the desired phase or gain margin. Three examples are given to show the effectiveness of the proposed controller.

Index Terms: time delay filter, process with dead time, deadbeat control, two-level control, control signal shaping

1 Introduction

Regulation is often the main task of most controllers used in industry, but in many cases it is also very important to obtain a fast response to set-point changes. These two control problems can be decoupled using the two degree-of-freedom (2DOF) techniques, see [1, 2, 3] for delay-free systems and [4, 5, 6] for delay systems, where a pre-filter is frequently used as the second degree-of-freedom to weight the set-point change so that the set-point response is desirable. Some pre-filters having particular properties have been studied in the literature. A variable set-point weighting scheme was proposed in [7], where adaptive techniques were used to adjust the set-point weighting. A deadbeat set-point response was obtained by using a time-delay filter in [8], where the reference signal for the closed-loop feedback system is converted to a pulse-step signal from the original step signal. An optimal control strategy, called pulse-step control, was proposed in [9] to obtain a settling time close to the time at which the impulse response reaches its maximum (when there is almost no input constraint). This is a feedforward law used with a proportional-integral-derivative (PID) feedback controller for regulation. The major drawback of the approach is that a set of nonlinear equations has to be solved to obtain the optimal switching times, which complicates the design. A minor drawback of the proposed controller is that it is only available for systems having a relative degree higher than 2, which means not directly applicable for most of the chemical processes modeled as a first-order plus dead-time (FOPDT) model.

An important issue when considering to obtain a fast response is that the input constraint has to be taken into account, at least to some extent. The approach proposed in [8] offers a deadbeat set-point response, which is very fast. However, there might be some problems in practice when using this control strategy. First of all, the first pulse in the converted set-point has a relatively large amplitude. This might cause the control signal saturated and the system performance degraded. Secondly, the set-point response was designed to reach the desired output in one sampling period, in addition to the inherent dead time. This might push the controller too hard and, intuitively, it might be difficult to obtain such a fast response. In this paper, we add more freedom to the controller and explicitly consider the controller saturation in design by shaping the control signal. The above-mentioned two drawbacks are avoided and hence the proposed control strategy is more practical. The response is still deadbeat and the deadbeat time is dependent on the maximal control output. A

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prominent property of the controller is that it provides a two-level control signal, which mimics experienced manual operations: the control outputs a value close to the maximal value to provide the largest acceleration for the process at first and then the control outputs a smaller value to keep the system output at the set point. This is done by designing the numerator of the feedback controller. The denominator of the feedback controller is designed to guarantee the system stability. In this paper, the feedback controller is designed to be a PI controller, of which the parameter can be freely tuned according to the desired gain or phase margin.

The advantages of the *two-level control* are:

- decoupled design for set-point and disturbance responses;
- fast deadbeat set-point response, no overshoot;
- easy tuning of the disturbance response;
- controller saturation is explicitly considered.

The rest of the paper is organized as follows. The controller is designed in Section 2 to shape the control signal to have two levels, to guarantee the stability of the closed-loop system and to obtain a good disturbance response. Three examples are given in Section 3 to show the design and the simulation results. Conclusions are made in Section 4.

2 Controller design

Consider the following FOPDT model:

$$G(s) = \frac{Ke^{-\tau s}}{Ts+1},$$

where K is the static gain, τ is the dead time and T is the apparent time constant. The control system under consideration is shown in Figure 1, where r' is the converted set point and e is the error signal (which is not $r - y$ here). The controller consists of a feedback controller $C(z)$ and a feedforward controller $F(z)$. Taking into account the sampling-hold effect, the generalized plant is

$$G(z) = K\frac{1-a}{z-a}z^{-l},$$

where $l = \tau/T_s$ is a positive integer and $a = e^{-T_s/T}$. Assume that the feedback controller is

$$C(z) = \frac{(1-az^{-1})N(z)}{D(z)}, \tag{1}$$

of which the order of the polynomials $N(z)$ and $D(z)$ in z^{-1} is n and m, respectively, then the closed-loop transfer function of the feedback system is

$$\begin{aligned} T_{yr}(z) &= F(z)\frac{C(z)G(z)}{1+C(z)G(z)} \\ &= F(z)\frac{K(1-a)N(z)z^{-(l+1)}}{D(z)+K(1-a)N(z)z^{-(l+1)}} \end{aligned} \tag{2}$$

and the transfer function from the set point $r(z)$ to the control signal u is

$$\begin{aligned} T_{ur}(z) &= F(z)\frac{C(z)}{1+C(z)G(z)} \\ &= F(z)\frac{(1-az^{-1})N(z)}{D(z)+K(1-a)N(z)z^{-(l+1)}}. \end{aligned} \tag{3}$$

It is not necessary to place a zero $z = a$ in the controller (1). However, this does not affect the explanation of the main idea in this paper and, as can be seen later, this does simplify the controller design and it is helpful for the system stability and the performance.

Simply, $F(z)$ can be chosen to cancel the closed-loop poles provided that the closed-loop system is stable. Hence, the function of $D(z)$, $N(z)$ and $F(z)$ is now clear: $D(z)$ can be designed to guarantee the stability of the closed-loop system;

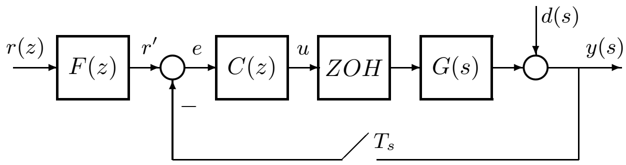


Figure 1: The control structure

$F(z)$ can be designed to obtain the desired set-point response and $N(z)$ can be chosen to *shape the control signal* as well as the set-point response.

The time-delay filter $F(z)$ can be designed to be

$$F(z) = \frac{D(z)}{K(1-a)} + N(z)z^{-(l+1)}. \tag{4}$$

As a result, the two transfer functions (2) and (3) become

$$T_{yr}(z) = N(z)z^{-(l+1)} \tag{5}$$

and

$$T_{ur}(z) = N(z)\frac{1-az^{-1}}{K(1-a)}. \tag{6}$$

Since the output y is expected to start just after the inherent dead time, $N(0)$ cannot be zero. As a result, $D(0)$ cannot be zero in order to guarantee the causality of the controller.

2.1 Shaping the control signal

In order to obtain a fast transient response, the expected control signal (with respect to a step set-point change) is shown in Figure 2. This is to mimic the experienced manual operations: the controller outputs the maximal u_{max} to drive the plant as hard as possible at the early stage and then the controller outputs a smaller value to keep the system output at the set point. This is called a *two-level* signal. Assume that the moment at which the change occurs is $(n+1)T_s$, then the form of this signal can be approximately obtained by the following transfer function (with respect to a unit-step set point):

$$\frac{1-a^{n+1}z^{-n-1}}{K(1-a^{n+1})}. \tag{7}$$

The amplitude of the first level of the control signal varies when the order of $N(z)$, i.e. n, changes. For a given u_{max}, the following condition has to be satisfied:

$$\frac{1}{K(1-a^{n+1})} \leq u_{max}, \tag{8}$$

which means

$$n \geq \frac{T}{T_s}\ln\frac{Ku_{max}}{Ku_{max}-1} - 1. \tag{9}$$

The smaller the value of the allowed u_{max}, the larger the value of n; the larger the ratio $\frac{T}{T_s}$, the larger the value of n. Hence, this parameter can be used to meet the requirement of the controller saturation. Surprisingly, n does not depend on the dead time of the plant.

According to (6) and (7), the desired $N(z)$ can be derived to be

$$N(z) = \frac{1-a^{n+1}z^{-n-1}}{1-az^{-1}} \cdot \frac{1-a}{1-a^{n+1}} = \frac{\sum_{i=0}^{n} a^i z^{-i}}{\sum_{i=0}^{n} a^i}. \tag{10}$$

Obviously, $N(z)$ satisfies the following constraint:

$$N(1) = 1,$$

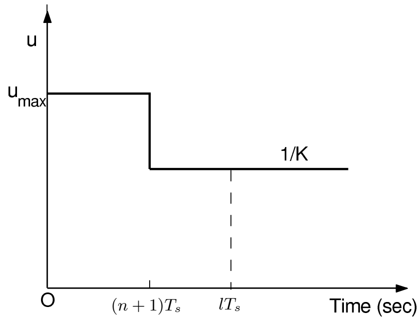


Figure 2: The desired control signal

which guarantees the zero steady-state error for the set-point response, according to (5). Moreover, the set-point response reaches the steady state at

$$(l+n+1)T_s \approx \tau + T \ln \frac{Ku_{max}}{Ku_{max}-1},$$

where the "$\approx$" is due to the approximate selection of n in (9). For a given plant, this time is independent of the controller and the sampling period and depends on the maximal controller output u_{max} only. In other words, this is an inherent characteristic of the system.

There is no *braking control* (a large negative action, which is common in the time-optimal control strategy) in the control signal. This is because: (1) there is no need for such a brake here because the response reaches the steady state in finite time and there is no overshoot; (2) the benefit of a large negative action is very small when u_{max} is not very large [9], which is the common case, and (3) the control strategy is more sensitive when there is a large negative control action [10].

2.2 Stability of the closed-loop system

There are many options to design the feedback controller, in particular $D(z)$, because the function of $D(z)$ is to guarantee the system stability. One simple option is to choose $D(z)$ to be an integrator, i.e.

$$D(z) = \frac{1-z^{-1}}{K_I},$$

where K_I is an integral coefficient such that the steady-state error with respect to step disturbances is 0. However, this brings a sluggish or oscillatory disturbance response. A better design is to choose

$$D(z) = \frac{1-z^{-1}}{K_I} N(z) \tag{11}$$

to obtain a better stability margin and a faster disturbance response. Another advantage by doing so is that the stability analysis and the parameter tuning are considerably simplified. This choice of $D(z)$ offers the following PI controller:

$$C(z) = \frac{(1-az^{-1})N(z)}{D(z)} = K_I \frac{1-az^{-1}}{1-z^{-1}}, \tag{12}$$

which is, by chance, almost the same as the one proposed in [8]. The corresponding open-loop transfer function is

$$L(z) = C(z)G(z) = \frac{K_I K(1-a)}{(z-1)z^l}. \tag{13}$$

A typical root locus of this system with respect to K_I is shown in Figure 3. Since $L(z)$ has no zero, all the $l+1$ loci approach asymptotically to $l+1$ straight lines, which start at

$$z = \frac{1}{l+1}$$

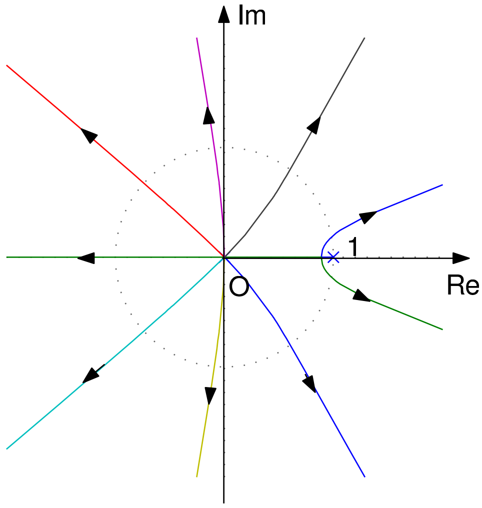


Figure 3: Typical root locus of the closed-loop system

with angles

$$\theta = \frac{2k+1}{l+1} \cdot 180^\circ, \qquad (k = 0, 1, 2, \cdots, l).$$

The $l+1$ loci can be categorized into two groups: one is the $l-1$ loci starting at the origin and then approaching ∞; the other is the two loci starting at $z = 0$ and $z = 1$, which meet together and then approach ∞ (if there is no zero $z = a$ in the controller, then the locus starting at the origin in the second group starts at $z = a$). Since the pole $z = a$ of the plant is canceled by the zero $z = a$ in the controller, the two loci closest to the right-half real axis are pushed towards the left-half plane. This means a larger gain margin or a faster disturbance response can be obtained. This is why such a zero is placed in the controller (1). Theoretically speaking, if one more zero is placed between $z = a$ and $z = 1$ then a larger gain margin can be obtained, but this conflicts with the requirement of the shape of the control signal. As can be seen from the root locus in Figure 3, there always exists a critical gain K_{Ic} such that only one real pole or a pair of complex poles arrive(s) at the unity circle and the others remain inside the unity circle. For any gain $0 < K_I < K_{Ic}$, the closed-loop system is always stable. Since the open-loop transfer function (13) is very similar to that obtained in [8], the stability lemma obtained there still holds and is cited below.

Lemma 1 [8] The closed-loop system designed above is stable if

$$0 < K_I < \frac{2}{K(1-a)} \sin \frac{\pi}{4l+2}.$$

In order to obtain a phase margin of ϕ_m, the integral coefficient K_I can be chosen as

$$K_I = \frac{2}{K(1-a)} \sin \frac{\pi - 2\phi_m}{4l+2}.$$

In order to obtain a gain margin of g_m, the integral coefficient K_I can be chosen as

$$K_I = \frac{2}{K(1-a)g_m} \sin \frac{\pi}{4l+2}.$$

Proof See [8].

When $n = 0$, we have

$$N(z) = 1,$$

which offers

$$T_{yr} = z^{-(l+1)}$$

and

$$T_{ur} = \frac{1 - az^{-1}}{K(1-a)}.$$

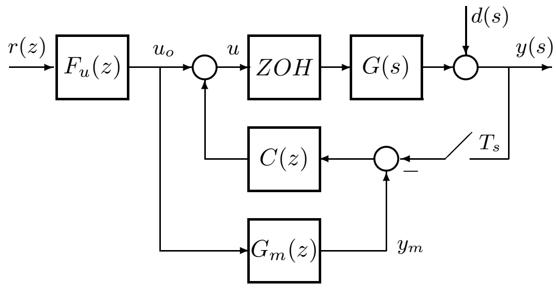


Figure 4: An alternative implementation

These responses are the same as the one obtained in [8]. According to (9) or (8), the maximal control signal should not be less than $\frac{1}{K(1-a)}$. Otherwise, the approach proposed in [8] makes the controller saturated and hence cannot be used.

2.3 An alternative implementation

The control system designed above can be implemented in an alternative structure shown in Figure 4, where the controller $C(z)$ is given in (12) and the filter $F_u(z)$ is

$$F_u(z) = \frac{1 - a^{n+1} z^{-n-1}}{K(1 - a^{n+1})}.$$

$G_m(z)$ is the model of the nominal plant, i.e.

$$G_m(z) = K \frac{1 - a}{1 - az^{-1}} z^{-(l+1)}.$$

The desired set-point response is

$$y_m(z) = G_m(z) F_u(z) r(z)$$

and, for $r(z) = 1$, it is

$$y_m(z) = N(z) z^{-(l+1)}.$$

This control structure, which was also discussed in [9], can be regarded as an open-loop controller $F_u(z)$ combined with a closed-loop controller $C(z)$. $F_u(z)$ is used to supply a desired control profile u_o while the desired set-point response y_m is supplied through an ideal model of the plant $G_m(z)$. $C(z)$ is designed to govern the disturbance response, the stability and the robustness.

There are some advantages to use this structure. It is clearer that the desired control signal can be designed in an open-loop way if the plant is stable. Another advantage of this implementation is that the controller design does not affect the shaping of the control signal, unlike the case discussed before (where $N(z)$ is a part of the controller). Hence, a different controller, e.g. a PID controller, can be easily designed. Using this structure, it is also possible to generalize the proposed idea to general high-order plants.

3 Examples

Three examples will be studied in this section. The first example is an FOPDT having a long dead time; the second one is an FOPDT having a short dead time; the third one is a multi-lag process which can be modeled as an FOPDT model. In the first example, the attention will be paid to the control signal and the set-point response. In the second example, the attention will be paid to the comparison of the disturbance response with respect to well-tuned PI controllers for both nominal and uncertain cases. In the last example, the attention will be paid to the effectiveness of the proposed method for multi-lag processes.

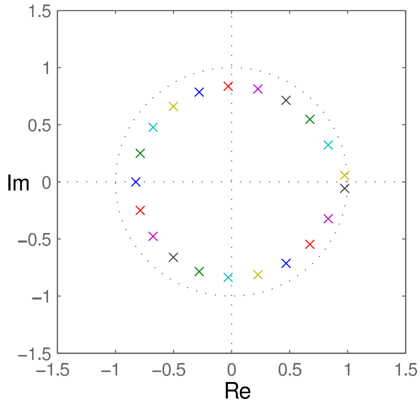


Figure 5: The distribution of the closed-loop poles

3.1 A process with long dead time

Consider the following FOPDT model having a long dead time:

$$G(s) = \frac{e^{-5s}}{s+1},$$

which was studied in [8]. Here, we choose the sampling period as $T_s = 0.25$ s. The parameters of the generalized plant are $a = 0.7788$ and $l = 20$. Assume that the upper bound of the controller output is $u_{max} = 1.45$, then

$$n \geq 3.68$$

We choose

$$n = 4$$

and hence

$$N(z) = 0.31 + 0.2414z^{-1} + 0.1881z^{-2} + 0.1464z^{-3} + 0.1141z^{-4}.$$

The parameter K_I is chosen as $K_I = 0.173$ to obtain a phase margin of $45°$. The corresponding distribution of the closed-loop poles is shown in Figure 5. All the 21 poles are inside the unity circle. The converted set point r' for $r = 1$ is shown in Figure 6. The set point has been changed into four pieces: the fi rst piece is a staircase signal larger than the original value due to the effect of $D(z)$ in $F(z)$; the second piece is 0 (this piece disappears if $m > l$); the third piece is another staircase signal due to the effect of $N(z)$ in $F(z)$; the last piece is the original set point. It is worth noting that the large magnitude in the fi rst piece, which is nothing else but just a value in the controller, does not cause the control signal saturated, see Figure 7. The control signal has two levels and stays below the upper bound. The system output reaches the steady state after $n + 1 = 5$ sampling steps, in addition to the inherent dead time of the plant, as shown in Figure 7. The error signal is shown in Figure 8. It is very clean. It remains 0 after m steps, even when the system output is increasing from 0 to the steady state. The disturbance responses with respect to $d = -0.2 \cdot 1(t - 15)$ for $K_I = 0.173$ as well as for $K_I = 0.115$, corresponding to a phase margin of $60°$, are shown in Figure 9. Bearing in mind, the change of K_I does not affect the set-point response and the corresponding control signal, although it does affect the converted set point and the control signal corresponding to the disturbance. Hence, the disturbance response can be freely tuned according to the requirement of the phase or gain margin.

3.2 A process with short dead time

Consider the following FOPDT model having a short dead time:

$$G(s) = \frac{e^{-0.5s}}{s+1},$$

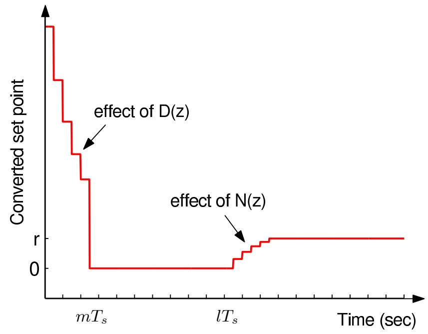


Figure 6: The converted set point

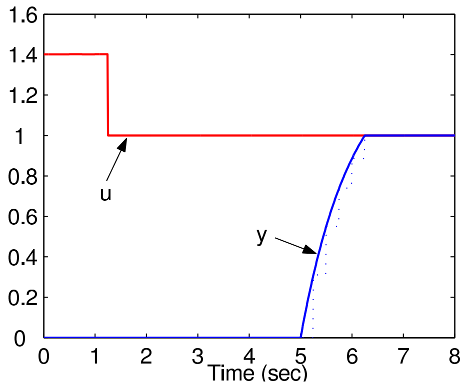


Figure 7: The output and the control signal

which was studied in [8, 11]. Here, we choose the sampling period as $T_s = 0.25$ s. The parameters of the generalized plant are $a = e^{-0.25}$ and $l = 2$. Assume that the upper bound of the controller output is $u_{max} = 1.45$, then

$$n \geq 3.68.$$

We choose

$$n = 4$$

and as a result,

$$N(z) = 0.31 + 0.2414z^{-1} + 0.1881z^{-2} + 0.1464z^{-3} + 0.1141z^{-4}.$$

As mentioned before, $N(z)$ does not depend on the dead time of the plant. The $N(z)$ obtained here is the same as the one obtained in the previous example.

K_I is chosen as 1.101 to obtain a phase margin of $55°$. The set point response and the corresponding control signal are shown in Figure 10. The control signal changes from the larger value to the smaller value at the $m = n+1 = 5$th step and the system output reaches the steady state at the $l + n + 1 = 7$th step. There is no oscillation in the control signal, no overshoot in the system output.

In the sequel, the simulation results are compared with two discrete-version PI controllers, of which the continuous ones $K_p(1 + \frac{1}{T_i s})$ are tuned by Zigler-Nichols method (noted as Z-N in figures) and Ho-Hang-Cao method [11] (noted as H-H-C in figures). The parameters tuned by Z-N method are $K_p = 1.8$ and $T_i = 1.5$ s and the parameters tuned by Ho-Hang-Cao method are $K_p = 1.05$ and $T_i = 1.0$ s to obtain a gain margin of 3dB and a phase margin of $60°$. All controller outputs in the simulations are bounded by $u_{max} = 1.45$. A step disturbance $d = -0.2$ acts at $t = 7$s. The control signals, the system outputs and the error signals in the nominal case are shown in Figure 11. The responding speeds with respect to the set-point changes are very close to each other, however, the response obtained by the proposed controller has no overshoot while the other two have about 20% overshoot. The settling time obtained by the proposed controller is much shorter than the other two. The excellent set-point response does not cause the degradation of the disturbance response.

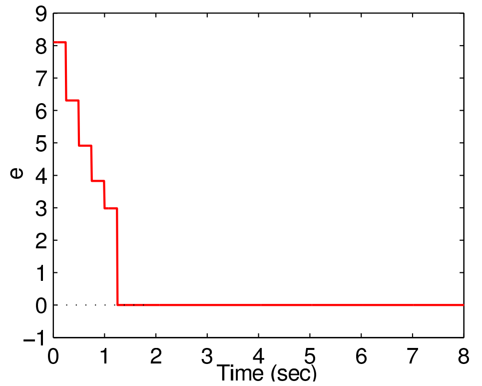


Figure 8: The error signal

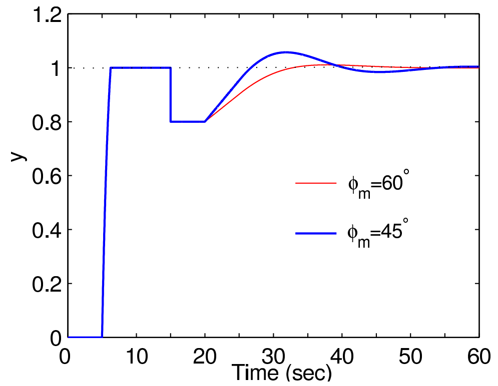


Figure 9: Disturbance responses

In fact, the disturbance response is still the best one (slightly better than that obtained by H-H-C). As to the control signal, there are only two levels in the control signal (in the part responding to the set-point change) of the proposed controller, but there are many levels in the other two. The proposed controller is not saturated but both the other two are saturated. A prominent property can be seen from the error signal shown in Figure 11 (c): the error signal e (see Figure 1) remains 0 after the effect of $D(z)$, i.e. after m steps, until there is a load disturbance. In other words, the proposed controller does not have error accumulation if there is no disturbance (and, of course, no uncertainty as well) but the other two controllers do have error accumulation. When there exists uncertainty in the plant, the proposed controller still behaves much better than the other two, as can be seen from Figure 12, where the responses in three cases having different uncertainties are shown.

3.3 A multi-lag process

Consider the following multi-lag process

$$G(s) = \frac{1}{(s+1)^5},$$

which was studied in [11]. Here, we design the controller using the following model:

$$G(s) = \frac{e^{-3s}}{2.73s+1}, \tag{14}$$

which is slightly different in that given in [11]. The sampling period is chosen as $T_s = 0.3$ s and the parameters of the generalized plant are $a = 0.8959$ and $l = 10$. Assume that the upper bound of the controller output is $u_{max} = 1.45$, then

$$n \geq 9.65.$$

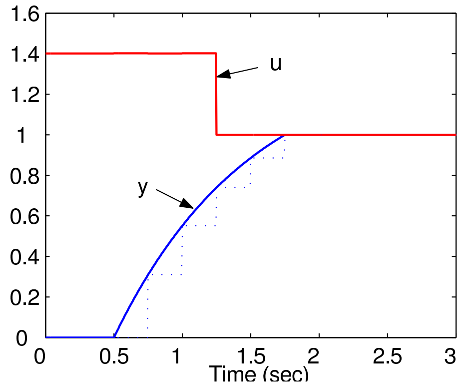


Figure 10: Set-point response and the control signal

We choose

$$n = 10$$

and as a result,

$$\begin{aligned} N(z) \quad = \quad & 0.1484 + 0.1329z^{-1} + 0.1191z^{-2} + 0.1067z^{-3} \\ & +0.0956z^{-4} + 0.0856z^{-5} + 0.0767z^{-6} + 0.0687z^{-7} \\ & +0.0616z^{-8} + 0.0552z^{-9} + 0.0495z^{-10}. \end{aligned}$$

The integral gain K_I is tuned as $K_I = 0.4791$ to obtain a phase margin of $60°$. The simulation results are shown in Figure 13 with comparison to those (noted by H-H-C in the figures) obtained by the PI controller $0.49(1 + \frac{1}{2.74s})$ tuned in [11] (to obtain a phase margin of $60°$ and a gain margin of 3dB). Since the proposed controller is designed according to the approximate model (14), the set-point response is no longer deadbeat. However, it is still much faster than the one obtained by the PI controller while the disturbance response is slightly better. The overshoot is also smaller. The control signals are quite different at the beginning: the proposed control signal outputs the maximal value from the beginning, but the PI controller has to integrate the error signal to reach the maximal value.

4 Conclusions

A new idea, *shaping the control signal*, has been proposed in this paper to offer *a two-level control* for processes with dead time and input constraints. The control signal (responding to the set-point change) consists of two levels: a large-value level and then a small-value level. The system output reaches the steady state in finite time, which is determined by the maximal control output and the plant parameters and is independent of the control parameters, even the sampling period. Hence, the sampling period can be freely chosen to obtain a satisfactory disturbance response. The disturbance response is governed by a PI controller, which can be tuned to guarantee a specified gain or phase margin. The proposed idea can be generalized to more general plants using other controllers, e.g. a PID controller. Three examples have shown that the *two-level control* indeed offers good set-point and disturbance responses. Further research on deadbeat control with input constraints for general plants will be undertaken.

References

[1] I.M. Horowitz, *Synthesis of Feedback Systems*, Academic Press, NY, 1969.

[2] D.C. Youla and J.J. Bongiorno Jr., "A feedback theory of two-degree-of-freedom optimal Wiener-Hopf design," *IEEE Trans. Automat. Control*, vol. 30, no. 7, pp. 652–665, 1985.

[3] M. Morari and E. Zafiriou, *Robust Process Control*, Prentice-Hall, Inc., 1989.

[4] Q.-C. Zhong and H.X. Li, “Two-degree-of-freedom PID-type controller incorporating the Smith principle for processes with dead-time,” *Industrial & Engineering Chemistry Research*, vol. 41, no. 10, pp. 2448–2454, 2002.

[5] L. Mirkin and Q.-C. Zhong, “Coprime parametrization of 2DOF controller to obtain sub-ideal disturbance response for processes with dead time,” in *Proc. of the 40th IEEE Conf. on Decision & Control*, Orlando, USA, December 2001, pp. 2253–2258.

[6] L. Mirkin and Q.-C. Zhong, “2DOF controller parameterization for time-delay systems,” submitted to IEEE Trans. AC, 2003.

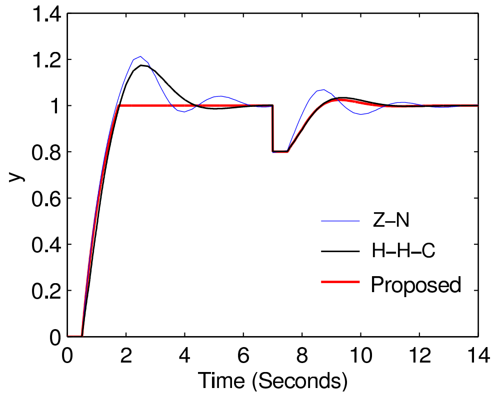
[7] C.C. Hang and L.S. Cao, “Improvement of transient response by means of variable set point weighting,” *IEEE Trans. Industrial Electronics*, vol. 43, no. 4, pp. 477–484, 1996.

[8] Q.-C. Zhong, J.Y. Xie, and Q. Jia, “Time delay filter-based deadbeat control of process with dead time,” *Industrial & Engineering Chemistry Research*, vol. 39, no. 6, pp. 2024–2028, 2000.

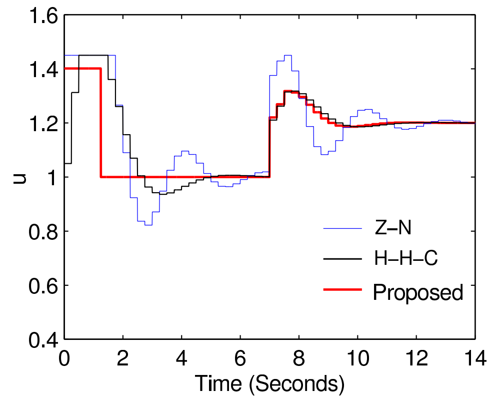
[9] A. Wallén and K.J. Åström, “Pulse-step control,” in *Proc. of the 15th IFAC World Congress*, Barcelona, Spain, 2002.

[10] K.J. Åström and K. Furuta, “Swinging up a pendulum by energy control,” *Automatica*, vol. 36, no. 2, pp. 287–295, 2000.

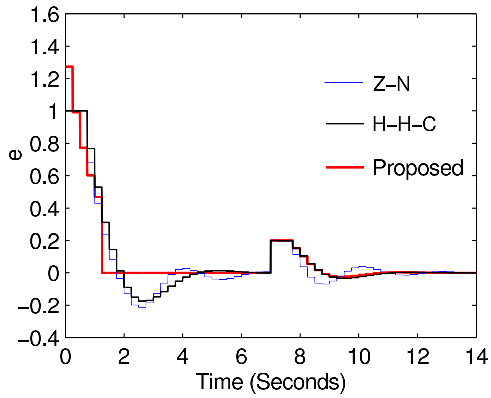
[11] W.K. Ho, C.C. Hang, and L.S. Cao, “Tuning of PID controllers based on gain and phase margin specifications,” *Automatica*, vol. 31, no. 3, pp. 497–502, 1995.



(a) the system outputs

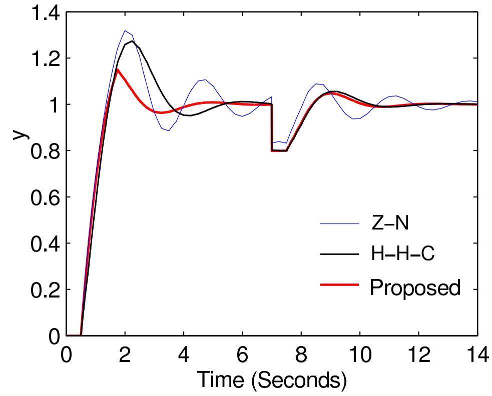


(b) the control signals

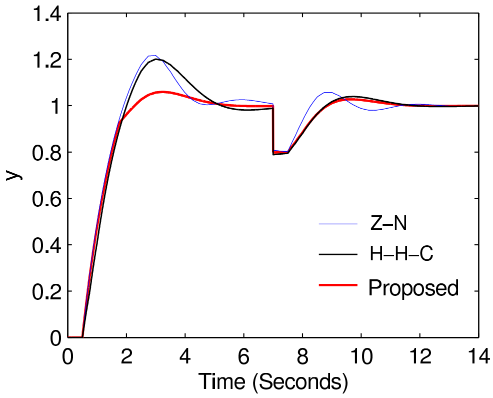


(c) the error signals

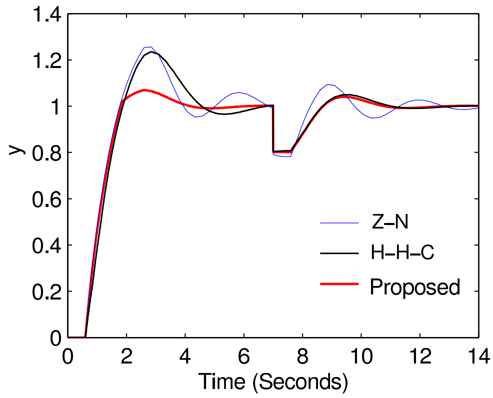
Figure 11: Response in the nominal case



(a) Case 1: K increased by 20%

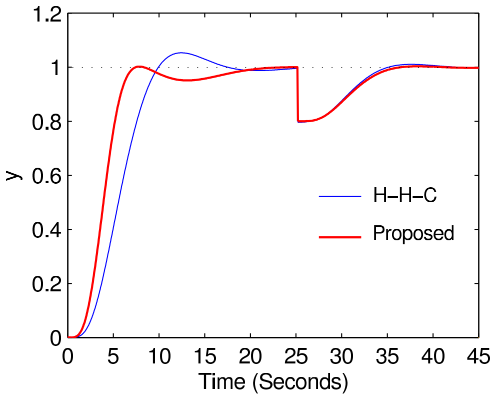


(b) Case 2: T increased by 20%

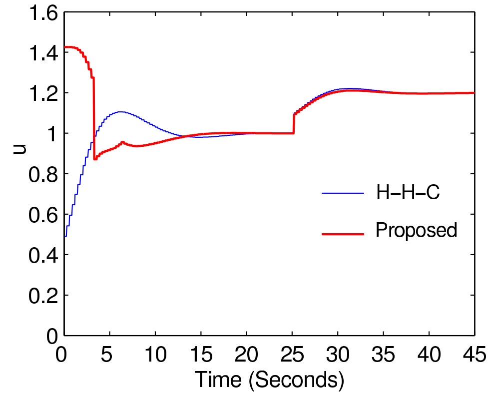


(c) Case 3: τ increased by 20%

Figure 12: Responses in three uncertain cases



(a) the system outputs



(b) the control signals

Figure 13: Responses of the multi-lag process