# Comments on
# “A Time Delay Controller for Systems with Uncertain Dynamics” *

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**Abstract**

The introduction of time delay control (TDC) has initiated a series of researches. However, the artificially-introduced delay brings difficulties into system analysis and causes some drawbacks. This paper presents an alternative method using an uncertainty and disturbance estimator (UDE) and shows that the delay element is not needed at all to obtain similar performances. The two inherent drawbacks of TDC (oscillations in the control signal and the need of measuring the derivative of the states) have also been eliminated. The robust stability of LTI-SISO systems is analyzed and simulations are given to show the effectiveness of the UDE control and the comparison with TDC. The conclusion is that there is no need of TDC in most cases.

**Keywords:** Time delay control (TDC), uncertainty and disturbance estimator (UDE), parametric uncertainty, robust control, interval plants

## 1 Introduction

About 15 years ago, Youcef-Toumi and Ito [1, 2] proposed a robust control method for systems with unknown dynamics, which is called time delay control (TDC). TDC is based on the assumption that a continuous signal remains unchanged during a small enough period and hence the past observation of uncertainties and disturbances can be used to modify the control action directly, rather than to adjust controller gains like gain scheduling or to identify system parameters like adaptive control. During the last decade, TDC has been widely studied, see e.g [3, 4, 5] and the references therein. It has been applied to DC servo motors [6], four-wheel steering systems [7], hybrid position/force control of robots [8], brushless DC motors [9], overhead cranes [10], robot trajectory control [11] and so on.

However, TDC inherently requires that all the states *and* their derivatives be available for feedback. This impose very strict limitations on the applications. Another inherent drawback of the TDC is that oscillations always exist in the control signal. As is well known in the control community, delay is not good for control. In particular, it brings difficulties into system analysis and tends to destabilize the system. Although it has been shown in the above-cited papers that TDC has very good potential to improve system performances, it is still desirable to get rid of delays, if possible, so as to simplify system analysis.

The assumption used in TDC is a time-domain assumption. In this paper, we use an assumption in the frequency domain to propose an alternative control strategy to obtain similar performances as TDC. The major part of the controller is called an uncertainty and disturbance estimator (UDE). The two inherent drawbacks of TDC disappear and the delay disappears, too. There is no need to measure the derivative of the states and there is no oscillation in the control signal. The system stability is easy to be analyzed. In addition to all these advantages, the system performance is very similar to that can be obtained by TDC. Hence, there is no need of TDC in many cases.

In order to keep clear comparison with the paper by Youcef-Toumi and Ito [2], a similar structure as the paper is used for this paper. In Section 2, the general UDE control law for uncertain systems is derived. More aspects of the control

*K. Youcef-Toumi and O. Ito, *Journal of Dynamic Systems, Measurement, and Control.* vol 112, pp.133-142, 1990.

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strategy are analyzed in Section 3 for LTI-SISO systems. The control strategy is then applied to two examples studied in the paper [2] in Section 4. The advantages of the proposed control strategy are very clear with comparison to TDC. Finally, conclusions are made in Section 5.

# 2 UDE-based Control Law

## 2.1 System description

The system to be considered is formulated as:

$$\dot{\mathbf{x}} = (\mathbf{A} + \mathbf{F})\mathbf{x} + \mathbf{B}\mathbf{u}(t) + \mathbf{d}(t), \tag{1}$$

where $\mathbf{x} = (x_1, ..., x_n)^T$ is the state vector, $\mathbf{u}(t) = (u_1(t), ..., u_r(t))^T$ is the control input, $\mathbf{A}$ is the known state matrix, $\mathbf{F}$ is the unknown state matrix, $\mathbf{B}$ is the control matrix having full column rank and $\mathbf{d}(t)$ is the unpredictable external disturbances.

## 2.2 Reference model and structural constraint

Assume that the desired specifi cations can be described by the reference model

$$\dot{\mathbf{x}}_m = \mathbf{A}_m\mathbf{x}_m + \mathbf{B}_m\mathbf{c}(t), \tag{2}$$

where $\mathbf{c}(t) = (c_1(t), ..., c_r(t))^T$ is the set-point. The control objective is to make the state error $\mathbf{e}$ between the system and the reference model

$$\mathbf{e} \;\; = \;\; \begin{pmatrix} x_{m1} - x_1 & \cdots & x_{mn} - x_n \end{pmatrix}^T = \mathbf{x}_m - \mathbf{x}, \tag{3}$$

converges to zero. In other words, the error dynamics

$$\dot{\mathbf{e}} = (\mathbf{A}_m + \mathbf{K})\mathbf{e} \tag{4}$$

is stable, where $\mathbf{K}$ is called the error feedback gain matrix.

Combine (1), (2), (3), and (4), we obtain

$$\mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c}(t) - \mathbf{A}\mathbf{x} - \mathbf{F}\mathbf{x} - \mathbf{B}\mathbf{u}(t) - \mathbf{d}(t) = \mathbf{K}\mathbf{e}. \tag{5}$$

Then the control action $\mathbf{u}(t)$ is obtained as

$$\mathbf{u}(t) = \mathbf{B}^+ \left[\mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c}(t) - \mathbf{A}\mathbf{x} - \mathbf{F}\mathbf{x} - \mathbf{d}(t) - \mathbf{K}\mathbf{e}\right], \tag{6}$$

where $\mathbf{B}^+ = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T$ is the pseudo inverse of $\mathbf{B}$.

Since $\mathbf{u}(t)$ given in (6) is the least-square solution rather than the accurate solution, (4) and (5) will only be met under the following structure constraint:

$$\left[\mathbf{I} - \mathbf{B}\mathbf{B}^+\right] \cdot \left[\mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c}(t) - \mathbf{A}\mathbf{x} - \mathbf{F}\mathbf{x} - \mathbf{d}(t) - \mathbf{K}\mathbf{e}\right] = 0. \tag{7}$$

Obviously, if $\mathbf{B}$ is invertible, the above structural constraint is always met. If not, the choice of the reference model and the error feedback gain matrix is somewhat restricted. However, as shown in [2], the systems described in the canonical form meet this constraint. We assume that this constraint is satisfi ed in the sequel.

## 2.3 UDE and the control law

The control law in (6) can be represented in $s$-domain by using Laplace transform (assuming zero-initial states) as

$$\mathbf{U}(s) = \mathbf{B}^+ \left[\mathbf{A}_m\mathbf{X}(s) + \mathbf{B}_m\mathbf{C}(s) - \mathbf{K}\mathbf{E}(s) - \mathbf{A}\mathbf{X}(s)\right] + \mathbf{B}^+ \left[-\mathbf{F}\mathbf{X}(s) - \mathbf{D}(s)\right], \tag{8}$$

where the Laplace transformation is denoted by the corresponding capital letter. The fi rst part in (8) is known while the second part, denoted hereafter by $\mathbf{U}_d(s)$, includes the uncertainties and the external disturbance. According to the system dynamics (1), the second part $\mathbf{U}_d(s)$ can be rewritten as,

$$\mathbf{U}_d(s) = \mathbf{B}^+\left[-\mathbf{F}\mathbf{X}(s) - \mathbf{D}(s)\right] = \mathbf{B}^+\left[(\mathbf{A} - sI)\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s)\right]. \quad (9)$$

In other words, the unknown dynamics and the disturbances could be observed by the system states and the control signal. However, it cannot be used in the control law directly. TDC adopts an estimation of this signal by using a small delay in time domain [2]. Here, we use a different estimation strategy in the frequency domain.

Assume that $G_f(s)$ is a strictly proper low-pass filter with unity steady-state gain and broad enough bandwidth, then $\mathbf{U}_d(s)$ can be approximated by

$$\mathbf{UDE} = \mathbf{B}^+\left[(\mathbf{A} - sI)\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s)\right] \cdot G_f(s). \quad (10)$$

This is called the uncertainty and disturbance estimator (UDE). The UDE only uses the control signal and the states to observe the uncertainties and the unpredictable disturbances. Hence,

$$\begin{aligned}\mathbf{U}(s) &= \mathbf{B}^+ \cdot \left[\mathbf{A}_m\mathbf{X} + \mathbf{B}_m\mathbf{C} - \mathbf{K}\mathbf{E} - \mathbf{A}\mathbf{X}\right] + \mathbf{UDE} \\ &= \mathbf{B}^+ \cdot \left[\mathbf{A}_m\mathbf{X} + \mathbf{B}_m\mathbf{C} - \mathbf{K}\mathbf{E} - \mathbf{A}\mathbf{X}(1 - G_f) - sG_f\mathbf{X} + G_f\mathbf{B}\mathbf{U}\right].\end{aligned}$$

The UDE control law is then derived as

$$\mathbf{U}(s) = (I - \mathbf{B}^+\mathbf{B}G_f)^{-1}\mathbf{B}^+ \cdot \left[\mathbf{A}_m\mathbf{X} + \mathbf{B}_m\mathbf{C} - \mathbf{K}\mathbf{E} - \mathbf{A}\mathbf{X}(1 - G_f) - sG_f\mathbf{X}\right]. \quad (11)$$

The control signal is formed by the current states $\mathbf{X}(s)$, the low-pass filter, the reference model and the error feedback gain. It has nothing to do with the uncertainty and the disturbance. Since $G_f$ is strictly proper, there is no need of measuring the derivative of states ($sG_f$ is physically implementable).

Assume that the frequency range of the system dynamics and the external disturbance is limited by $\omega_f$, then the ideal low-pass filter $G_f(s)$ has a low-frequency gain ($\omega < \omega_f$) of 1 and a high-frequency gain ($\omega > \omega_f$) of zero. Hence, the estimation error of the uncertainty and the disturbance

$$\mathbf{B}^+\left[(\mathbf{A} - sI)\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s)\right](1 - G_f)$$

is zero for all the frequency range because $1 - G_f = 0$ for the low-frequency range and $(\mathbf{A} - sI)\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s) = 0$ for the high-frequency range. A practical low-pass filter is

$$G_f(s) = \frac{1}{Ts + 1}, \quad (12)$$

where $T = 1/\omega_f$. Although this will cause some error in the estimation, the steady-state estimation error is still zero because $G_f(0) = 1$ (the initial value of $G_f(s)$ can always be chosen to be zero).

So far, the analysis is based on linear systems. However, this control strategy can also be generalized to nonlinear systems with unknown dynamics, using similar expositions as in [2]. In fact, all the simulations will be done for nonlinear systems.

## 3 Special Case: LTI-SISO systems

In this Section, we consider LTI-SISO systems with uncertainties and disturbances described in the canonical form, i.e. the corresponding matrices have the following partitions:

$$\mathbf{A} = \begin{pmatrix} 0 & \mathbf{I}_{n-1} \\ \multicolumn{2}{c}{\mathbf{A}_1} \end{pmatrix}; \mathbf{F} = \begin{pmatrix} 0 \\ \mathbf{F}_1 \end{pmatrix}; \mathbf{B} = \begin{pmatrix} 0 \\ b \end{pmatrix}; \mathbf{d}(t) = \begin{pmatrix} 0 \\ d(t) \end{pmatrix}, \quad (13)$$

where $\mathbf{A}_1 = (-a_1, -a_2, ..., -a_n)$ and $\mathbf{F}_1 = (-f_1, -f_2, ..., -f_n)$ are $1 \times n$ row vectors, and $\underline{f}_i \leq f_i \leq \overline{f}_i$ $(i = 1, \cdots, n)$ are uncertain parameters. $\mathbf{I}_{n-1}$ is the $(n-1) \times (n-1)$ identity matrix and $b \neq 0$. The output of the system is assumed to be $y = x_1$.

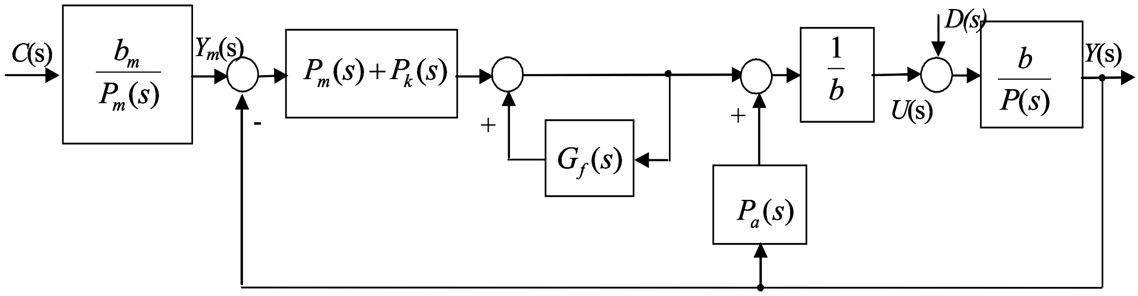



Figure 1: The equivalent structure of UDE-controlled LTI-SISO systems

## 3.1 Control scheme

The reference model and the error feedback gain matrix are partitioned as

$$\mathbf{A}_m = \begin{pmatrix} 0 & \mathbf{I}_{n-1} \\ & \mathbf{A}_{m1} \end{pmatrix}; \mathbf{B}_m = \begin{pmatrix} 0 \\ b_m \end{pmatrix}; \mathbf{K} = \begin{pmatrix} 0 \\ \mathbf{K}_1 \end{pmatrix}, \tag{14}$$

where $\mathbf{A}_{m1} = (-a_{m1}, -a_{m2}, ..., -a_{mn})$ and $\mathbf{K}_1 = (-k_1, -k_2, ..., -k_n)$ are $1 \times n$ row vectors.

Substitute the matrices into (11), the control law is

$$U(s) = \frac{1}{b\left[1 - G_f(s)\right]}\left[-\sum_{i=1}^{n} a_{mi}X_i + b_mC + \sum_{i=1}^{n} k_iE_i - sG_fX_n\right] + \frac{1}{b}\sum_{i=1}^{n} a_iX_i. \tag{15}$$

It can be further simplifi ed as follows:

$$\begin{aligned} U(s) &= \frac{1}{b\left[1 - G_f(s)\right]}\left[-\sum_{i=1}^{n}(a_{mi} + k_i)X_i + b_mC + \sum_{i=1}^{n} k_iX_{mi} - s^nG_f(s)X_1\right] + \frac{1}{b}\sum_{i=1}^{n} a_iX_i \\ &= \frac{1}{b\left[1 - G_f(s)\right]}\left[-\sum_{i=1}^{n}(a_{mi} + k_i)s^{i-1}X_1 + b_mC + \sum_{i=1}^{n} k_is^{i-1}X_{m1} - s^nG_f(s)X_1\right] + \frac{1}{b}\sum_{i=1}^{n} a_iX_i \\ &= \frac{1}{b\left[1 - G_f(s)\right]}\left\{-\left[P_m(s) + P_k(s)\right]Y + b_mC + P_k(s)\frac{b_m}{P_m(s)}C + s^n\left[1 - G_f(s)\right]Y\right\} + \frac{1}{b}\sum_{i=1}^{n} a_is^{i-1}X_1 \\ &= \frac{P_m(s) + P_k(s)}{b\left[1 - G_f(s)\right]}\left[\frac{b_m}{P_m(s)}C - Y\right] + \frac{s^n}{b}Y + \frac{1}{b}\sum_{i=1}^{n} a_is^{i-1}Y \\ &= \frac{P_m(s) + P_k(s)}{b\left[1 - G_f(s)\right]}\left[\frac{b_m}{P_m(s)}C - Y\right] + \frac{P_a(s)}{b}Y \end{aligned} \tag{16}$$

where $G_m(s) = \frac{b_m}{P_m(s)} = \frac{b_m}{s^n + a_{mn}s^{n-1} + \cdots + a_{m1}}$ is the transfer function of the reference model, $P_a(s) = s^n + \sum_{i=1}^{n} a_is^{i-1}$ is the characteristic polynomial of the known system dynamics and $P_k(s) = k_ns^{n-1} + \cdots + k_1$ is the error feedback polynomial.

The equivalent structure of the UDE control scheme is shown in Figure 1. The UDE controller can be divided into four parts: the reference model $\frac{b_m}{P_m(s)}$ to generate a desired trajectory; a local positive-feedback loop via the low-pass fi lter (behaving like a PI controller); the polynomial $P_m(s) + P_k(s)$ for the tracking error and the polynomial $P_a(s)$ for the output. The last two perform the derivative effect. Obviously, UDE control uses more derivative information than the common PID controller and hence UDE control has the potential to obtain better performances than the common PID controller. It is worth noting that this structure is the *equivalent* one of UDE-based control, but not the structure for implementation. The control law should be implemented according to (15) under the assumption that all the states are available for feedback.

### 3.2 Stability analysis

The transfer function of the LTI-SISO plant given by (13) is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b}{s^n + (a_n + f_n)s^{n-1} + \cdots + (a_1 + f_1)} = \frac{b}{P(s)}. \tag{17}$$

From (16), the closed-loop transfer function of the system is derived to be

$$\frac{Y(s)}{C(s)} = \frac{b_m}{P_m(s)} \cdot \frac{P_m + P_k}{P_m + P_k + (1 - G_f)(P - P_a)}. \tag{18}$$

Since the reference model is always chosen to be stable, only the stability of the second part in the above equation is to be discussed hereafter. If the low-pass filter is chosen as given in (12), then the characteristic polynomial of the closed-loop system is,

$$\begin{aligned} P_c(s) &= (Ts + 1)(P_m + P_k) + Ts(P - P_a) \\ &= (P_m + P_k) + Ts(P_m + P_k + P - P_a) \end{aligned} \tag{19}$$

Obviously, the stability depends on the reference model, the error feedback gain, the time constant of the filter and, of course, the plant itself. The reference model is chosen to obtain the desired specifications and the time constant of the low-pass filter is chosen to have broad enough bandwidth. Hence, the error feedback gain **K** can be used to gurantee the robust stability of the closed-loop system. Intuitively, if $T$ is small enough, the system is always stable. However, this might cause the problem of implementation and hence the value of $T$ has to be compromised. The robust stability of the polynomial (19) can be guaranteed by the following theorem, which is obtained directly using the box theorem [12] (a generalization of the well-known Kharitonov's theorem [13]) for parametric uncertain systems.

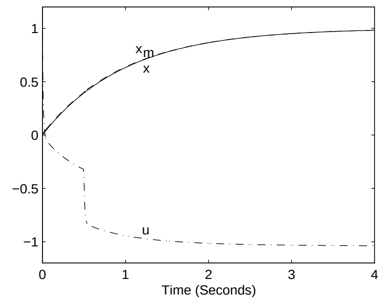
**Theorem** (Robust stability) The closed-loop system is stable iff the 4 polynomials $P_m(s)+P_k(s)+TsP_{pq}(s)$ ($p, q = 1, 2$) are stable, in which $P_{pq}(s) = P_p(s) + Q_q(s)$ ($p, q = 1, 2$) are the Kharitonov polynomials of the uncertain polynomial $P_m + P_k + P - P_a = s^n + \sum_{i=1}^{n}(a_{mi} + k_i + f_i)s^{i-1}$ with

$$\begin{aligned}
P_1(s) &= (\beta_1 + \underline{f}_1) + (\beta_3 + \overline{f}_3)s^2 + (\beta_5 + \underline{f}_5)s^4 + \cdots \\
&= \sum_{i=0,even}^{n} (\beta_{1+i} + j^i \cdot \min\left\{j^i \underline{f}_{1+i}, j^i \overline{f}_{1+i}\right\}) \cdot s^i, \\
P_2(s) &= (\beta_1 + \overline{f}_1) + (\beta_3 + \underline{f}_3)s^2 + (\beta_5 + \overline{f}_5)s^4 + \cdots \\
&= \sum_{i=0,even}^{n} (\beta_{1+i} + j^i \cdot \max\left\{j^i \underline{f}_{1+i}, j^i \overline{f}_{1+i}\right\}) \cdot s^i, \\
Q_1(s) &= (\beta_2 + \underline{f}_2)s + (\beta_4 + \overline{f}_4)s^3 + (\beta_6 + \underline{f}_6)s^5 + \cdots \\
&= \sum_{i=1,odd}^{n} (\beta_{1+i} + j^{i-1} \cdot \min\left\{j^{i-1} \underline{f}_{1+i}, j^{i-1} \overline{f}_{1+i}\right\}) \cdot s^i, \\
Q_2(s) &= (\beta_2 + \overline{f}_2)s + (\beta_4 + \underline{f}_4)s^3 + (\beta_6 + \overline{f}_6)s^5 + \cdots \\
&= \sum_{i=1,odd}^{n} (\beta_{1+i} + j^{i-1} \cdot \max\left\{j^{i-1} \underline{f}_{1+i}, j^{i-1} \overline{f}_{1+i}\right\}) \cdot s^i,
\end{aligned}$$

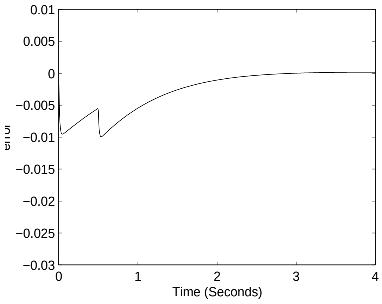
where $\beta_i = a_{mi} + k_i$ ($i = 1, \cdots, n$), $\beta_{n+1} = 1$, $\underline{f}_{n+1} = \overline{f}_{n+1} = 0$ and $j = \sqrt{-1}$.

## 4 Examples

In order to show clear comparisons with TDC, we use two examples studied in [2].




(a) the output and the control signal




(b) the tracking error

Figure 2: The nominal response

## 4.1 A first-order nonlinear system

Consider the following first-order nonlinear system,

$$\dot{x} = f \cos x + u + d$$

where, $0.5 \leq f \leq 2$ is an uncertain parameter with the nominal value $f = 1$. The nominal system is the same as the one in [2] and $d = 0.5 \cdot 1(t - 0.5)$ is a step disturbance.

The reference model is chosen as $\dot{x}_m = -x_m + c$ and the error feedback gain is $k = 0$. The low-pass filter is $G_f(s) = \frac{1}{Ts+1}$ with $T = 0.01s$. According to (11), the control law is

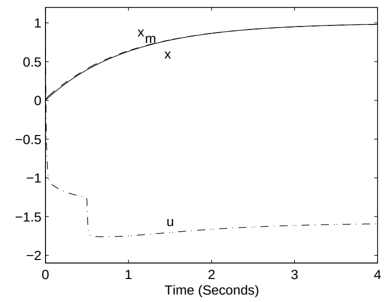
$$U(s) = \frac{1}{1 - G_f(s)} \left[ -X(s) + C(s) - sG_f(s)X(s) \right].$$

The measurement of $\dot{x}$ is not needed. The command signal is $c(t) = 1(t)$ in the simulations. The nominal response is shown in Figure 2 and the response when $f = 2$ is shown in Figure 3. Comparing with the responses in [2], the performance is much better. Although the uncertain parameter $f$ is changed to $f = 2$, the performance is not degraded too much. Another advantage is that there is no oscillation in the control signal of UDE-controlled system but there are oscillations in that of the TDC system.

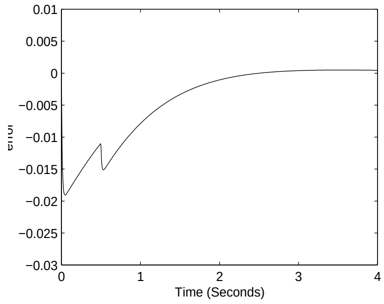
The bandwidth of the low-pass filter is very important for the system performance. Three cases with different time constants are shown in Figure 4. The smaller the time constant, the broader the bandwidth and the better the performances of both trajectory and disturbance-rejection. However, the time constant might be limited by the computation capability and the measurement noise in practice.

## 4.2 A servo motor positioning system

Consider the servo motor positioning system studied in [2]. The inherent unknown dynamics are viscous and dry frictions. Additional uncertainty is simulated by an elastic spring that is physically attached to the motor load. The dynamic equation of the system is




(a) the output and the control signal




(b) the tracking error

Figure 3: Response when $f = 2$

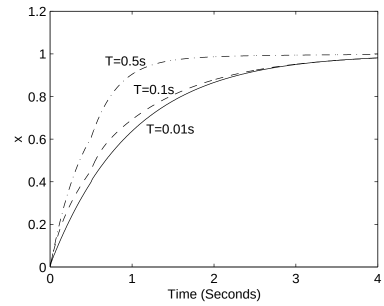



Figure 4: Effect of the time constant on the performance

$$\ddot{\theta} = -\frac{k_s}{J}\theta - \frac{b_s}{J}\dot{\theta} - d_s sgn(\dot{\theta}) + \frac{1}{J}\tau, \tag{20}$$

where $\theta$ is the motor rotation angle, $\tau$ is the motor torque, $k_s$ is the unknown spring constant, $b_s$ is the unknown viscous friction coefficient, $d_s$ is the unknown dry friction coefficient, and $J = 4.0 \times 10^{-4} kg \cdot m^2$ is the total inertia.

The reference model is chosen as the following second order system

$$\begin{pmatrix} \dot{\theta}_m \\ \ddot{\theta}_m \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega \end{pmatrix} \begin{pmatrix} \theta_m \\ \dot{\theta}_m \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_n^2 \end{pmatrix} c, \tag{21}$$

where $\omega_n = 5rad/s$ and $\zeta = 1$ are the same as those in [2] and the command $c = 1rad$ . The error feedback gain is chosen to be 0. The UDE control law is

$$\tau(s) = \frac{4.0 \times 10^{-4}}{1 - G_f(s)} \left[ -25\theta(s) - 10\dot{\theta}(s) + 25C(s) - sG_f(s)\dot{\theta}(s) \right], \tag{22}$$

where the low-pass filter $G_f(s)$ is $\frac{1}{Ts+1}$ with $T = 0.005s$. Obviously, the angular acceleration signal is not needed for feedback control, unlike the case of time delay control [2]. In order to compare with TDC, simulations are also done using the following TDC control law given in [2]:

$$\tau(t) = \tau(t-L) + J \cdot \left[ -25\theta(t) - 10\dot{\theta}(t) + 25c(t) - \frac{\dot{\theta}(t) - \dot{\theta}(t-L)}{L} \right],$$

where $L = 0.005s$.

#### 4.2.1 Nominal performance

When the frictions are omitted and the spring is not attached to the motor load, $k_s = 0$, $b_s = 0$, $d_s = 0$. The responses are shown in Figure 5. They are almost the same as the reference model under the UDE control or time delay control. There are oscillations in the control signal of time delay control. In order to see the oscillations clearly, the control signals of both cases are re-drawn in Figure 6.

#### 4.2.2 Robust performance

In this case, we assume that $k_s = 0.17Nm/rad$, $b_s = 0.01Nms/rad$ and $d_s = 0.1s^{-1}$. The responses are shown in Figure 7. They are all degraded because the time delay or the time constant is not small enough. It can be seen that the two control signals are different. There are oscillations in the case of TDC while the UDE control signal is very smooth. When the time constant or time delay is chosen as $0.001s$, the responses match the reference response very well and are almost not affected by the uncertainties, as shown in Figure 8.

## 5 Conclusions

This paper presents an uncertainty and disturbance estimator (UDE) for systems with uncertainties and disturbances. Similar performance can be obtained without the use of a delay element. Moreover, the inherent drawbacks of the TDC has been eliminated. There is no need of measuring the derivative of the states and there is no oscillation in the control signal. It is also easier to analyze the stability of a UDE control system than to analyze the stability of a TDC control system. In a word, there is no need of time delay control in most cases. However, this does not mean that the research work proposed in [2] is not important. As mentioned in the Introduction, TDC has initiated a series of studies, including the studies reported in this paper. Further work should be done to show if the delay is not needed either for the input-output linearization using time delay control, as reported in e.g. [14, 15].

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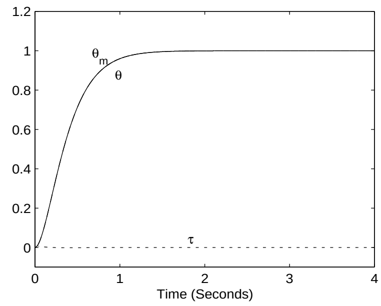
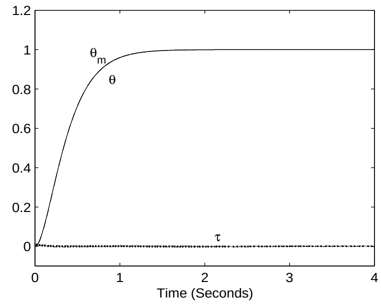
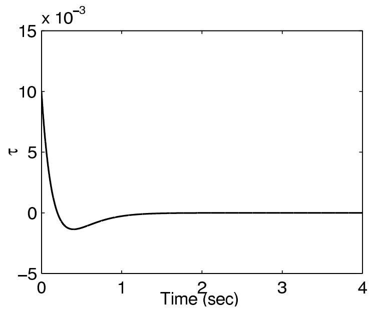



(a) UDE control

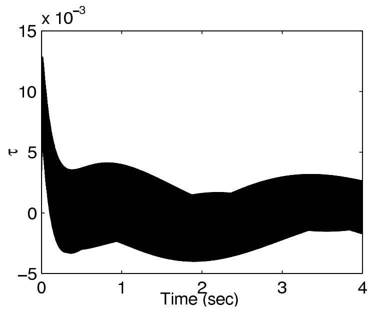



(b) Time delay control

Figure 5: Nominal response

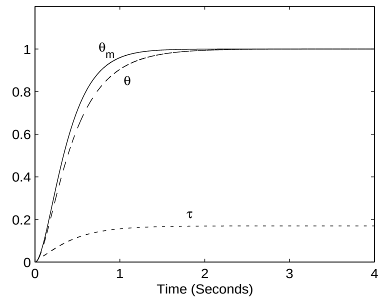



(a) UDE control

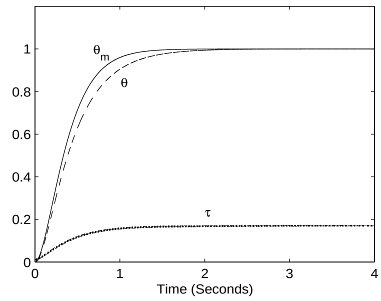



(b) Time delay control

Figure 6: The control signals

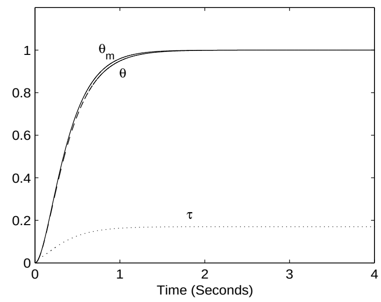



(a) UDE control

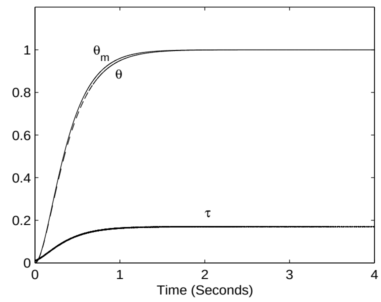



(b) Time delay control

Figure 7: Robust performance




(a) UDE control




(b) Time delay control

Figure 8: Robust performance when $L = 1ms$ or $T = 1ms$

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