Bridging Finite-Spectrum Assignment and Smith Predictor

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***Abstract*— This note establishes the bridge between two celebrated control strategies for processes with dead time: (generalized) Smith predictor (SP) and finite spectrum assignment (FSA). It is shown that they are two equivalent representations of all stabilizing controllers for processes with dead time. This finding may shed new light on the research in time-delay systems.**

Index Terms: Dead-time systems, finite-spectrum assignment, Smith predictor, observer-predictor, predictor-observer

I. Introduction

Smith predictor and finite-spectrum assignment (FSA) are two celebrated control strategies for processes with dead time. The significance of Smith predictor, invented in 1957 by O.J.M. Smith [1], is that a control problem for processes with dead time has been converted to a problem for the corresponding delay-free processes. SP was originally presented for stable processes with dead time. It was then extended to unstable processes with dead time, called generalized Smith predictor, see [2], [3], [4]. In the sequel, Smith predictor (SP) refers to the generalized Smith predictor. Very recently, it has been shown that (generalized) Smith predictor also plays an important role in H_∞ control of delay systems, see e.g [5], [6], [7], [8], [9]. Finite-spectrum assignment [10], [11], [12] was originally presented to overcome the drawback of Smith predictor (not applicable to unstable processes) by introducing a finite-interval integral over the past controls into the control law. It has been extended to more general cases, see [13], [4], [14], [12] and the references therein.

However, these two approaches have been regarded as independent, e.g., FSA is not mentioned in [3] and the authors of [13] commented "FSA provides not only an alternative way but also certain advantages over the Smith predictor" in the preface. No link between them has been established in the literature. This note shows that these two control strategies are nothing else but two equivalent representations of stabilizing controllers for processes with dead-time.

II. Smith predictor: Predictor-observer representation

Assume that the process to be controlled is $P(s)e^{-sh}$, of which the delay-free part has the following minimal realization:

$$P(s) = \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right].$$

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Just before submitting this note to IFAC Workshop TDS'03 and IEEE TAC, Dr. Mirkin told me that he had found this fact before. He did not include it in [15] even he did not mention FSA there, but he mentioned it as a Remark in the journal version of [15], which has not yet been published. Hence, although my finding is independent, it will not be submitted for publication.

Let F and L be such that $A+BF$ and $A+LC$ are Hurwitz. Then all stabilizing controllers [15], incorporating a generalized Smith predictor

$$Z(s) = Ce^{-Ah}(I - e^{-(sI-A)h}) \cdot \left[\begin{array}{c|c} A & B \\ \hline I & 0 \end{array}\right],$$

can be parameterized as shown in Figure 1, where

$$J(s) = \left[\begin{array}{c|cc} A+BF+e^{Ah}LCe^{-Ah} & -e^{Ah}L & B \\ \hline F & 0 & I \\ -Ce^{-Ah} & I & 0 \end{array}\right]$$

and $Q(s)$ is any stable transfer matrix.

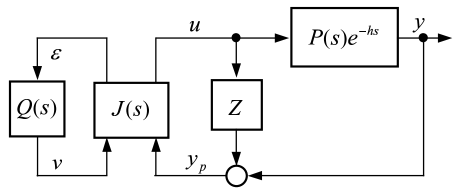

Fig. 1. Stabilizing controllers for processes with dead time

Denote the state vector of J by x_J, then

$$u = Fx_J + v,$$
$$\varepsilon = -Ce^{-Ah}x_J + y_p.$$

The state equation is given by

$$\dot{x}_J = (A+BF+e^{Ah}LCe^{-Ah})x_J - e^{Ah}Ly_p + Bv$$

or equivalently by

$$e^{-Ah}\dot{x}_J = (A+LC)e^{-Ah}x_J - Ly_p + e^{-Ah}Bu.$$

Using the above formulae, all the stabilizing controller shown in Figure 1 can be represented as shown in Figure 2. It consists of an output predictor Z and a state observer for the delay-free system $\left[\begin{array}{c|c} A & B \\ \hline I & 0 \end{array}\right]$ (because $x_J = \left[\begin{array}{c|c} A & B \\ \hline I & 0 \end{array}\right] u$).

III. Finite-spectrum assignment: Observer-predictor representation

The foundation of the finite-spectrum assignment is the state feedback using the predicted state of the process [11], [12]. The predicted state of the process $P(s)e^{-sh}$ is

$$x_p(t) = e^{Ah}x(t) + \int_{-h}^{0} e^{-A\zeta}Bu(t+\zeta)d\zeta, \qquad (1)$$

and the finite-spectrum assignment control law is given by

$$u(t) = \tilde{F}e^{-Ah}x_p(t),$$

see [11]. The Laplace transformation of the integral in (1) is the output of the following FIR block activated by u:

$$Z_x(s) = (I - e^{-(sI-A)h}) \cdot \left[\begin{array}{c|c} A & B \\ \hline I & 0 \end{array}\right].$$

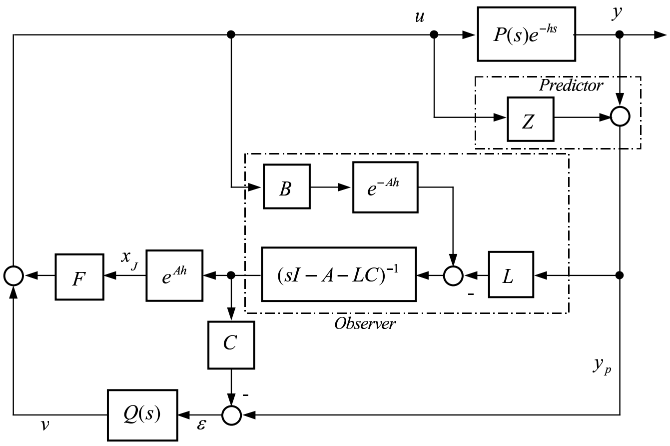

Fig. 2. SP: Predictor-observer structure

If the state $x(t)$ is not available for prediction, then a Luenberger observer is required to re-construct the state from the output y and the control u. It is easy to check that

$$\left[\begin{array}{c|c} A+LC & I \\ \hline I & 0 \end{array}\right] \cdot (Be^{-sh}u - Ly) = \left[\begin{array}{c|c} A & B \\ \hline I & 0 \end{array}\right] e^{-sh} \cdot u$$

gives the observed state x_o. Using the above formulas, the FSA control structure (using output feedback) can then be depicted in Figure 3 (where $F = \tilde{F}e^{-Ah}$ and $Q(s) = 0$). It consists of a state observer and a state predictor. As a matter of fact, this is exactly the central controller given in Figure 1, see [15]. It is also easy to check that ε can be written as

$$\varepsilon = Cx_o + y$$

so that all the stabilizing controller given in Figure 1 can also be represented as the observer-predictor structure shown in Figure 3. This is the FSA version of the stabilizing controllers.

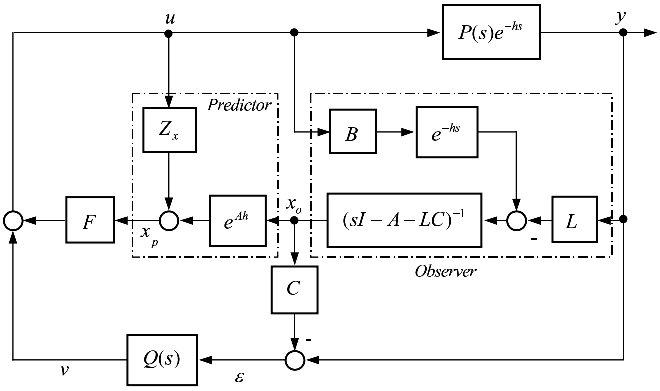

Fig. 3. FSA: Observer-predictor structure

IV. Discussions and conclusions

It has been shown that the FSA scheme and the SP scheme are theoretically equivalent. The two schemes are very similar to each other. Roughly speaking, only the order of the predictor and the observer is exchanged: in FSA schemes, the observer goes first and then the predictor but, in SP schemes, the predictor goes first and then the observer. The only change from the FSA scheme to the SP scheme is to change/move Z_x to Z and e^{-sh} in the observer to e^{-Ah}.

Some more insightful observations are: (i) the predictor in SP schemes is an *output predictor* while the one in FSA schemes is a *state predictor*; (ii) the observer in SP schemes is a state observer of the *delay-free* system while the one in FSA schemes is a state observer of the *delay* system (and hence a state predictor has to be used before state feedback); (iii) the free parameter $Q(s)$ might be useful for improving the performance of FSA schemes; (iv) since, in general, the predictors have to be approximately implemented, the robustness with respect to the implementation error has to be analyzed. From this point of view, further researches have shown that SP schemes have better robustness than FSA schemes [16]. Hence, a control law designed using the FSA scheme is suggested to convert to a SP structure for implementation. An intuitive explanation for this is that the observer in SP schemes attenuates the implementation error but the observer in FSA schemes does not; (v) in the both cases, the central controller is intrinsically a state feedback controller, using the *predicted* state (x_p in FSA schemes) or the *observed* state of the delay-free system (x_J in SP schemes).

Thanks to the bridge established in this note, the results obtained in both the FSA framework and the SP framework can be re-considered in the unified framework. This observation may throw new light on the study of time-delay systems.

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