Multibody System Dynamics **6:** 123–142, 2001.

A Motorcycle Model for Stability and Control Analysis

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(Received: 8 November 1999; accepted in revised form: 27 April 2000)

Abstract. The observed dynamic behaviour of motorcycles suggests that interesting and significant motions occur that are not currently understood. The most elaborate modelling exercise completed so far has produced results that need confirmation and extension. The construction of these models necessitates the use of automated methods and one such modelling methodology is described. The automated model building platform that was used here is AutoSim. This code was used to generate a variety of linear and nonlinear models in symbolic form. The relatively complex geometry of the steering system and the front tyre force system is discussed in detail and a new method of checking the self-consistency of the model is described and exploited. Sample results in the form of root-locus plots for small perturbations from straight running and cornering equilibrium states are presented. These are used to reproduce important findings from the literature. Conclusions are drawn on the basis of the results presented.

Key words: motorcycle, stability, control, dynamics.

1. Introduction

Motorcycles have several interesting dynamic properties, some of which are already well understood. For example, anybody who has ridden a bicycle will realise that above a certain low speed, the unstable roll (or capsize) instability disappears as the speed increases. Once this stable speed has been reached, the machine then displays lightly-damped oscillatory behaviours that are well understood in the straight running case. The modes of oscillation are commonly called wobble and weave and may be initiated by even small perturbations from straight running [1–3]. These motions are only a relatively simple subset of the general machine motions. There is growing evidence to suggest that more general motions are of interest from rider safety and machine design points of view.

The linear theory of the small perturbation motions near to straight running is well developed [1] and quite well validated by experiments [4–12]. The essential ingredients of the theory are: (a) separate bodies for the front and rear frame that are joined via an inclined steering head; (b) spinning road wheels; (c) lateral translation, yaw and roll freedoms of the rear frame; (d) twist and steer freedoms of the front frame relative to the rear frame; (e) a fairly elaborate steady-state tyre force and moment system representation that may well be empirical in nature; (f) the lag mechanisms by which tyre forces are delayed with respect to the slip phenomena that produce them; (g) some aerodynamic effects, mainly so that tyre loads can be properly adjusted as the motorcycle speed alters; and (h) some freedom for the rider's upper body to roll relative to the rear frame of the vehicle. The motions of major interest occur at substantially constant speed, so that the forward speed of the machine and the spin velocities of the road wheels may be constrained. When the tyres are rolling without significant longitudinal slip, the tyre force system is somewhat simplified, as compared with the completely general case.

As compared with the straight running case, there are more complex motions associated with small perturbations from an equilibrium cornering condition. In the cornering case, the motorcycle's forward speed, yaw rate, lateral acceleration and lean angle must be held at constant values. It should be noted that the geometry of the machine, the tyre force system and the aerodynamic forces all change with the lean angle. Also, what were previously decoupled lateral and vertical motions now become interactive. It therefore becomes essential to include main frame bounce, pitch and suspension freedoms in addition to the modelling ingredients listed above. The coupling of the in-plane and out-of-plane motions was discussed by Jennings [13] and shown in detail by Koenen [14], whose work demonstrated the analytical complexity of the theory at this level. We believe that this type of cornering study is at the limit of practicality for hand analysis methods. Despite one's best efforts to eliminate errors, the detailed accuracy of this kind of model is bound to remain suspect. The symbolic multi-body software system, AutoSim [15], has been applied recently to the general motorcycle cornering problem by Gani [16]. A small part of his achievement was the accurate recreation of the hand derived results of Sharp [2]. This model is a fore-runner of the present model.

Advancing from the above small perturbation scenario, large geometry motions and forcing from road irregularities, which causes time variation of parameter values, demand the inclusion of non-linear terms in the equations of motion. The system dynamics contain new possibilities and there is mounting evidence from machine usage that these issues require serious study. In the sequel we describe the setting up of a model for the general motion of a motorcycle, using AutoSim [15]. The use of the model to find the trim cornering states and the linearised equations of motion for small perturbations about those states is described. Most of the results included are intended for direct comparison with the corresponding studies in [14]. This work may promote changes in design methods for high-performance motorcycles. Some brief discussion of the full non-linear and time varying problem with road profile forcing is included but a detailed study of these issues is for the future.

2. Fundamentals of AutoSim

AutoSim [15] is a language used to derive the equations of motion of general multi-body systems and is based on the object oriented language LISP [17]. The syntactic rules of AutoSim are straightforward. The output from AutoSim takes one of several forms: (a) a Rich Text Format file containing the symbolic equations of motion of the system described; (b) a 'C' or 'FORTRAN' language simulation program with appropriate data files containing parameter values and simulation run control parameters; or (c) linear state-space equations in a MATLAB 'M-file' format [18, 19]. It should be noted that AutoSim's linearisation of the non-linear equations of motion is symbolic and completely general. It is possible to determine the non-linear steady turning equilibrium state and all the parameters required to describe fully a small perturbation dynamics problem. The multi-body basis of AutoSim is Kane's equations [20], an expression of the virtual power or Jourdain principle [21], which deals equally easily with holonomic and non-holonomic constraints. The simulation programs written by AutoSim contain many software engineering features which help to reduce execution times.

On occasions AutoSim expressions are used to explain the motorcycle modelling theory and so it is important for the reader to pay some attention to the AutoSim programming language itself. At the core of the language are commands like add-body, add-point, add-speed-constraint, add-line-force, add-strut, add-moment, no-movement, etc., which, to some extent, are self-explanatory. Such commands contain argument lists that are required to make their meaning precise. An AutoSim program begins by defining an inertial reference frame `n`, with fixed origin, `n0`, and fixed vector directions `[nx]`, `[ny]` and `[nz]`. As new bodies are added, having freedoms relative to `n`, local origins and axes are employed. All the reference axis systems introduced are right-handed Cartesian sets and inertias are normally referenced to axes parallel to these through the material body's mass centre. AutoSim can transform from local co-ordinates to global ones and vice-versa. Specifically, points specified globally are conveniently used to define points in bodies. The body-fixed points coincide with the corresponding global points, fixed in `n`, when the system is in its nominal configuration.

There are several commands for defining and manipulating vectors. For example, `pos(p1,p2)` defines a vector that points from `p2` to `p1`; `dot(vel(p1),[nz])` is the component of the absolute velocity of `p1` in the direction `[nz]`; `dir(pos(p1,p2))` is a unit vector in the direction `p2` to `p1`; `dplane([rwy],[nz])` is the projection of `[rwy]` onto the plane normal to `[nz]`; and `cross(a,b)` is the vector product of `a` and `b`. In each case, the square brackets indicate a unit vector along a (Cartesian) body axis. The 'setsym' command is used to define a new symbol, such a symbol being identified by the '@' character.

At the head of an AutoSim program are some commands that are used to reset the system, define the gravitational field, select the unit system to be employed, specify the simulation language in which the code written by AutoSim is to be

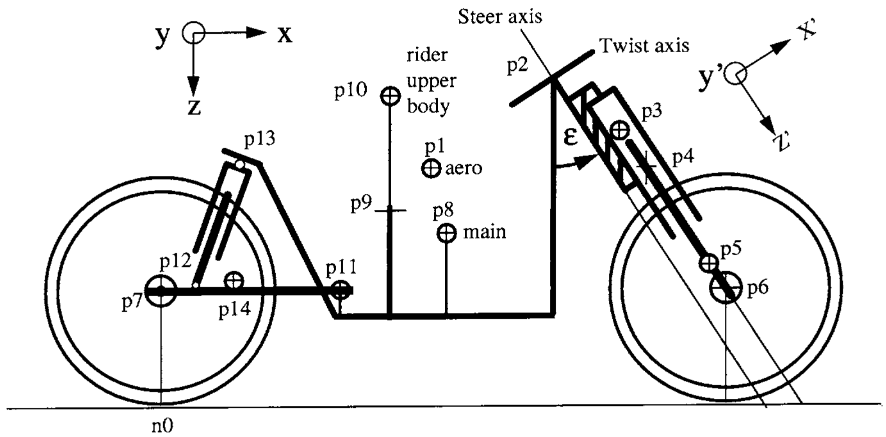

Figure 1. Motorcycle model in its nominal state with the key points labelled.

generated, select the type of numerical integrator to be installed in the code and to control the symbolic manipulations in various other ways. When the analyst does not exercise choice overtly in these matters, AutoSim uses default settings from a configuration file that the analyst is free to access.

3. Modelling the Motorcycle

The important parts of the motorcycle model are shown in Figure 1. The geometry depicted here is that for the nominal configuration in static equilibrium and the key points of the model are labelled. Prior work [6, 14, 22, 23] has shown that the compliance of the rear frame is important, because it impacts on the dynamic properties of the front frame via the boundary conditions. Other sources of compliance in the frame are known to be less important [24] and are consequently ignored here.

The multi-body structure and the relative freedoms between the bodies are shown in Figure 2. For completely general motions of the motorcycle, the wheel spin freedoms would be left unconstrained with the rotations determined by a tyre force model. In this study we concentrate on a more constrained set of possibilities in which the longitudinal tyre forces are relatively small and the acceleration or deceleration of the machine is modest. In these circumstances, the lateral and longitudinal tyre force system interactions are small and can be neglected. In this case an overt longitudinal tyre force description can be avoided by constraining the wheels to rotate without longitudinal slip. The effect of such constraints is to include in the calculations the gyroscopic torques due to the spinning wheels.

Due to the geometrical complexity in the region of the front tyre to ground contact, which is further discussed in Section 3.1 and to which the kinematic analysis of the Appendix relates, a precise description of the no-slip condition at the front wheel leads to complex symbolic expressions, which AutoSim [15] will deal with. Indeed, the fact that AutoSim will deal with these details is evidence

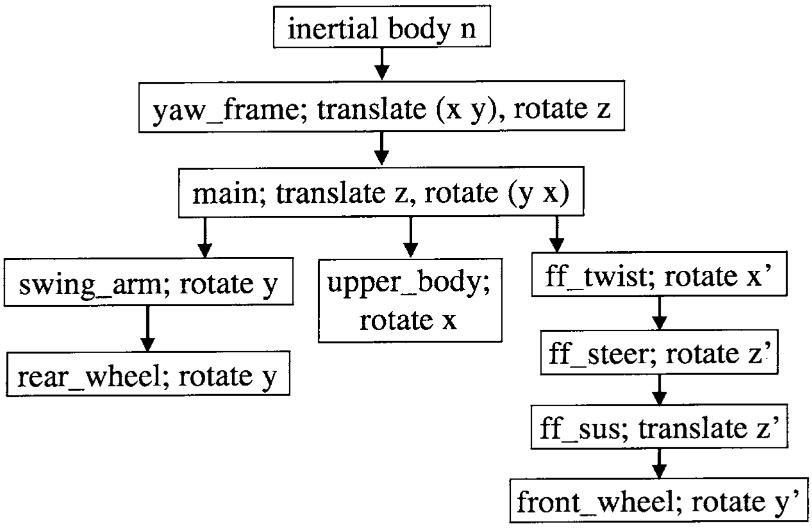

Figure 2. Body structure diagram, showing the parent/child relationships and relative freedoms allowed.

of its manipulative computational power. The effect of this precise but complex description is to marginally alter the magnitude of the gyroscopic torque due to the spin of the front wheel. As a result, it is appropriate to approximate these effects when dealing with the no-slip condition associated with the front wheel spin.

Adding forces to represent the effects of suspension springs and dampers and aerodynamics is straightforward. Similarly, moments are added to deal with the rider-upper-body-lean torque relative to the rear frame, the torque due to twisting the frame in the region of the steering head and aerodynamic effects. In order to describe the tyre forces and moments, it is necessary to consider the geometry of the system treated in some detail. The front frame geometry is relatively complex.

3.1. SOME GEOMETRIC DETAILS

Each wheel/tyre combination is treated as a thin disc with a radial flexibility. In its most physical form, the tyre has a massless outer ring making point contact with the ground. This outer ring can translate in the direction from the contact point to the wheel centre, relative to the massive part of the wheel. Corresponding to the translation described, the compliant material of the tyre sidewall is deflected and an appropriate spring restoring force is developed. When the motorcycle is in its nominal configuration, the tyre is loaded, the material strained and there is a force built into the sidewall structure. This force can be calculated from the static equilibrium conditions. Any change in deflection from the nominal is accompanied by a force change that is easy to describe. Such a laterally rigid tyre does not yield a realistic relaxation behaviour, so this is imposed by including conventional

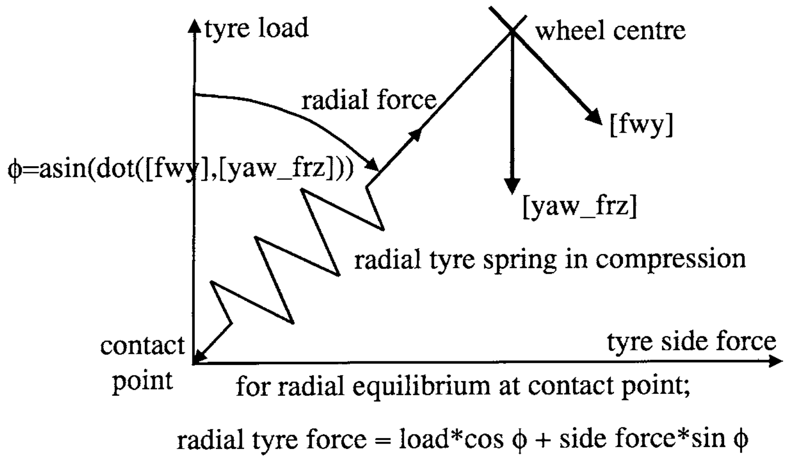

Figure 3. The tyre loading showing a radial deformation of the structure and a mutual dependency of vertical load and sideforce. View from rear.

first order lag equations to relate tyre sideforce and aligning torque to sideslip and camber angles; see Section 3.2.

The tyre loading is illustrated in Figure 3 in which it can be seen that the vertical load on the tyre will influence the tyre sideforce via the tyre force model. The sideforce will in turn affect the vertical load through the equilibrium requirements of the massless outer ring. Due to the lagging of the tyre lateral forces described above and further in Section 3.2, the apparent circular dependency is avoided in the model. The side forces for the current instant in time follow from the past history of these forces via the first order lag equations.

The geometry of the front frame and wheel is shown in some detail in Figure 4. The wheel camber angle is found as the arcsine of the vertical component of the wheel spindle unit vector `asin(dot([fwy],[yaw_frz]))`. The lateral direction unit vector, `fw_lat`, is defined from the projection of the wheel spindle unit-vector onto the ground plane `dir(dplane([fwy],[yaw_frz]))` and the longitudinal unit vector, `fw_long`, is set perpendicular to both the lateral and vertical directions `dir(cross(@fw_lat,[yaw_frz]))`. The vertical component of the vector joining the origin of the yaw frame to the front wheel centre is the height from the ground of the wheel centre. Dividing this by the camber angle gives the distance from wheel centre to the ground contact point. In the nominal condition, this distance is the wheel radius, so the tyre radial deflection from the nominal can be found by subtraction. This deflection is converted into a force change via the tyre radial stiffness. Adding the nominal state force gives the tyre radial force. The direction joining the wheel centre to the contact point is perpendicular to the wheel spindle (the normal to the wheel plane) and the long direction, so the unit vector can be set using the *vector* product `cross(@fw_long,[fwy])`. The magnitude and direction of the line joining wheel centre to contact point are next combined to give

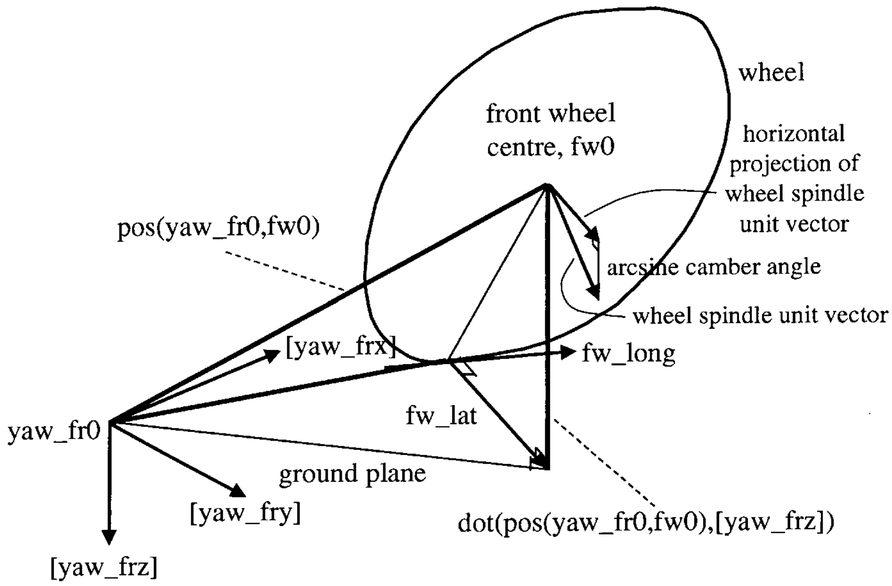

Figure 4. Front wheel and tyre geometry, showing method for describing ground contact point.

the vector: `dot(pos(yaw_fr0,fw0),[yaw_frz])*cross(@fw_long,[fwy])/cos(@phif)`, which is then resolved in the wheel axis directions `[fwx]` and `[fwz]` to define the coordinates of the contact point in the wheel system: `dot(@fwf_vec,[fwx])` and `dot(@fwf_vec,[fwz])`. The contact point `(fwcp)` is defined by its coordinates as a moving point in the wheel. This point is used to calculate the sideslip angle and it is the point of application of the load and the sideforce.

The sideslip angle is the arcsine of the velocity of the contact point in the lateral direction divided by the magnitude of its velocity `asin(dot(dir(vel(fwcp)),@fw_lat))`. The load, the slip angle and the camber angle are converted to tyre forces and moments according to Koenen's tyre model [14]; see Section 3.2.

In other work [16], we have preferred to use Sakai's results [25] as a basis for this and more recently, Magic Formula methods have been applied to the motorcycle tyre [26, 27], with the possibility of improving on previous methods. One of the immediate objectives is to reproduce Koenen's motorcycle results [16] using essentially his model and parameter values and to test the technical precision of his analysis. For this narrow purpose, his model is clearly the best, so we adopt it somewhat uncritically. In the case of the real tyre, the contact point moves round the sidewall as the wheel cambers, giving rise to an overturning moment. This effect is not reproduced in the thin disc tyre assumed, so the overturning moment is calculated separately and added. This calculation is illustrated in Figure 5.

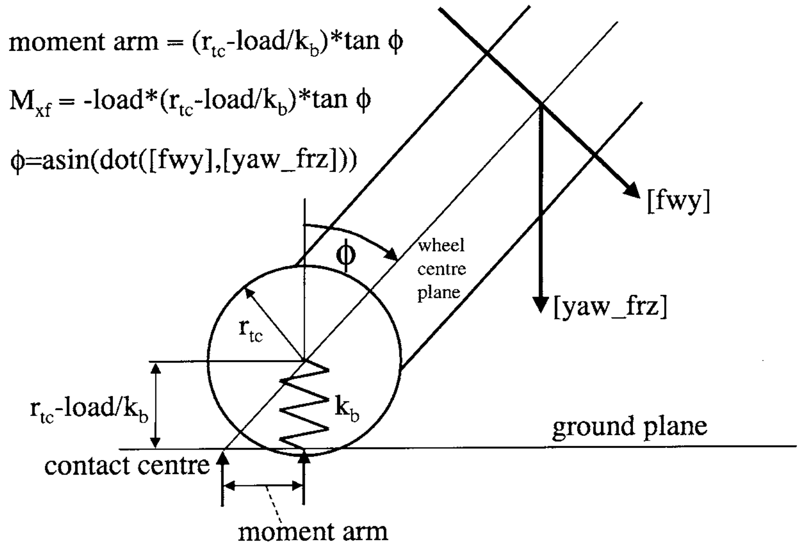

Figure 5. Rounded profile of a typical motorcycle tyre, showing how the overturning moment, M_{xf}, arises and how it is computed.

3.2. TYRE FORCE AND MOMENT DESCRIPTIONS

According to Koenen [16], the tyre produces a side force in response to side-slip and camber angles, an aligning moment in response to side-slip and camber angles and an overturning moment in response to camber angle. Koenen also included turn-slip in his model but found the influences negligible, so we have omitted that mechanism here. Force responses to side-slip are lagged through a conventional first-order lag relation and the aligning moment from slip is similarly lagged. Aligning moments from camber and overturning moments are assumed to be instantaneous.

Tyre force coefficients, cornering stiffnesses, camber stiffnesses, the pneumatic trail, relaxation lengths and so on for the nominal state, together with their load sensitivities, are required data. The camber sensitivity of the cornering stiffness is also accounted for. With the cornering stiffness adjusted for load and camber changes from nominal, the steady state side force is then taken to be proportional to the slip angle. No slip saturation mechanism is accommodated, on the basis that the slip angles will always be small enough for this to be reasonable. A similar description for the side force due to camber is provided but the camber influence on the camber stiffness, in this case makes for a parabolic dependence of force on angle.

The aligning moment response to side-slip involves representing the pneumatic trail as linearly decreasing as the absolute value of the slip angle increases and being proportional to the square root of the tyre load to the nominal load ratio. The aligning moment due to camber is proportional to camber angle via a coefficient that decreases linearly with the absolute camber angle and is proportional to the load ratio to the power 2.5. The overturning moment was described earlier. The relaxation length used in the lag equations is taken to be proportional to the square root of the load ratio. The precise relations used can be observed in the program listing at http://www.ee.ic.ac.uk/control/motorcycles/.

3.3. CONTROL OF SPEED AND STEADY TURNING LEAN ANGLE BY RIDER

Simple control systems are used to establish the required values for the forward speed and lean angle. The speed controller is a simple proportional-integral scheme that acts on the speed error to produce a torque that acts on the rear wheel from the main frame. The control gains were found by simple trial and error techniques. A cornering manoeuvre can be enforced via a lean angle controller. The lean angle controller has proportional-integral-derivative terms which operate on the error between a reference lean angle and the actual lean angle to produce a steering torque. Again, this controller was tuned by trial. As one moves towards higher values of lean angle at low or high speeds, this tuning process becomes more difficult. It goes without saying that if the controller gains are set incorrectly, the simulation model may go unstable.

3.4. MOTORCYCLE DESIGN PARAMETER VALUES

The intention was to represent Koenen's machine [16] as closely as possible and as a consequence his parameter values were used whenever appropriate. Our parameters are specified in the AutoSim model file listing, in the '`(set-defaults)`' command. In this command, each symbol is followed by its numerical value in SI units. It must be said that some uncertainty exists in respect of Koenen's frame twist damping (Cp_twst) and rider upper body damping (Cp_ubr) parameters. These are derived from the specification of the damping factors of the relevant decoupled modes, 0.1 for the frame twist and 0.25 for the rider, using the single degree of freedom system formula: Coefficient = 2∗factor∗sqrt(stiffness∗mass). In Koenen's expressions, the square root is missing and it is unclear how the omission should be interpreted. We have tried both possibilities and concluded that the problem is likely to be one of documentation. The square-root is presumed present in our computation of the parameters Cp_twst and Cp_ubr.

4. Diagnostics and Accuracy Checking

It is obviously important to try and validate new theoretical results by checking them against experimental results. At the present time this is not possible in the case of motorcycle handling over a wide range of running conditions. The test track, instrumentation and rider requirements for such testing are considerable, implying substantial resources and some danger. It follows that all the reasonable challenges to a new and unproven model should be posed and the model should only be accepted as a basis for the prediction of behaviour if all challenges fail to reveal any errors, or inconsistencies with known phenomena. The first such check with a new model is to do with ensuring that the model built by the modelling system is the same as the one conceived by the analyst. This is considered first. Following that some deeper challenges are described.

After AutoSim load has built a model, one can perform a number of elementary checks to see if the model is as expected. To start, the positions of all the points in local and global coordinates, the orientations of all bodies, the forces and moments added and the coordinates and the selected generalised speeds can be listed. Best practice is to check the model at this elementary level, before checking the dynamic properties of the simulation. Our model has, of course, proven satisfactory in this respect.

The next stage in the checking process comes from the observation that motorcycle accelerations are relatively simple in the equilibrium turning (trim) state. All the bodies have virtually the same (centripetal) acceleration towards the centre of the turn, so that the 'inertia forces' can be given a vector description for this restricted set of conditions. This description has a large degree of independence from the full AutoSim model description. In the latter, much of the work that AutoSim carries out is to evaluate the full set of inertia forces, for quite general motions of the material system, according to Kane's equations. In the former, the analyst devises the vector expressions required and AutoSim merely evaluates them. 'Equilibrium' equations, having a large degree of independence from the original model, can then be constructed. These equations must be substantially satisfied by the state variable values and forces and moments yielded by the full simulation, when the motorcycle settles into a steady turn, trim condition. The checking process is to run the rider-stabilised model to a defined steady state turning condition and to find that state. The state variables are then provided as data to the equilibrium checking equations. If all is well, the force and moment summations all come to zero. The coding included towards the end of the program listing at http://www.ee.ic.ac.uk/control/motorcycles shows this process in full detail. Commands are included as follows:

1. Specify an inertia force '`-m*tu(yaw_fr,1)*ru(yaw_fr)` in the direction `[yaw_fry]` for each mass, *m*, at its mass centre. The expression `tu(yaw_fr,1)` is the forward speed, while `ru(yaw_fr)` is the motorcycle yaw rate. The vector from `yaw_fr0`, the origin at the nominal contact point of the rear tyre, to the mass centre is of the form "'`pos(mcmc,yaw_fr0)`'.
2. Specify a gravitational force '`m*g`' in direction `[yaw_frz]` on each mass at its mass centre.
3. Specify a moment of momentum '`Ifwy*ru(fw)*[fwy]`' with the front wheel spin and '`Irwy*ru(rw)*[rwy]`' with the rear wheel spin; `Ifwy` and `Irwy` are the wheel spin inertias.

4. Calculate the rates of change of these moments of momentum as ‘`cross(ru(yaw_fr)*[yaw_frz], Ifwy*ru(fw)*[fwy])`’ and ‘`cross(ru(yaw_fr)*[yaw_frz], Irwy*ru(rw)*[rwy])`’.
5. Write out the sum of the lateral tyre forces, the lateral aerodynamic lift force and the lateral inertia forces.
6. Write out the sum of the vertical tyre forces, the vertical aerodynamic lift force and the gravitational forces.
7. Calculate the sum of the moments of forces and moments themselves about `yaw_fr0`. Include front tyre vertical and lateral forces; both aligning moments; both overturning moments; the aerodynamic drag and lift forces; the inertia forces and the gyroscopic moments (the negatives of the rates of change of moment of momentum).
8. Output the scalar components of the total moment of force about `yaw_fr0`.

Zero outputs in stages 5, 6 and 8 indicate a match (for the steady turn) between the full inertia force system for general conditions worked out by AutoSim and the simplified version for steady-state cornering described above. Again, the code has been developed to the point where these checks have proved successful.

5. Results

The main results to be included are root-locus plots for the nominal motorcycle in straight running and cornering at trim lean angles of 0.4 and 0.8 rad. Also, the influences of suspension damping changes on the loci at 0.8 rad lean are studied. Koenen [14] included each of these plots and, in respect of suspension damping effects, his results were noticeably in conflict with the experimental results of Weir and Zellner [8], the statements of Jennings [13] and general experience. It is vital to check the results of our model against those of Koenen. In addition, it is of interest to study the suspension damper influences, since a conflict of evidence currently exists on this issue of practical importance.

We began by checking that the diagnostics outlined above were operating satisfactorily. The next stage in the generation of results is to set up a process for solving the general steady turning equilibrium problem. This can be done by repeatedly running the C program written by AutoSim to the desired equilibrium condition. A preferred alternative is to carry out a low acceleration, or deceleration rate simulation run in which the machine passes slowly through the full range of speeds of interest. During this process, the desired roll angle is maintained by the steering controller action. Good controller settings were found by trial, enabling the (simulated) machine to stably settle into the desired steady-state. This is particularly difficult at high roll angles and at the ends of the speed range. The controllers need tuning anew for each lean angle. In effect, the nonlinear differential equations together with the speed controller combine to provide an algorithm for solving the nonlinear algebraic steady-state equations.

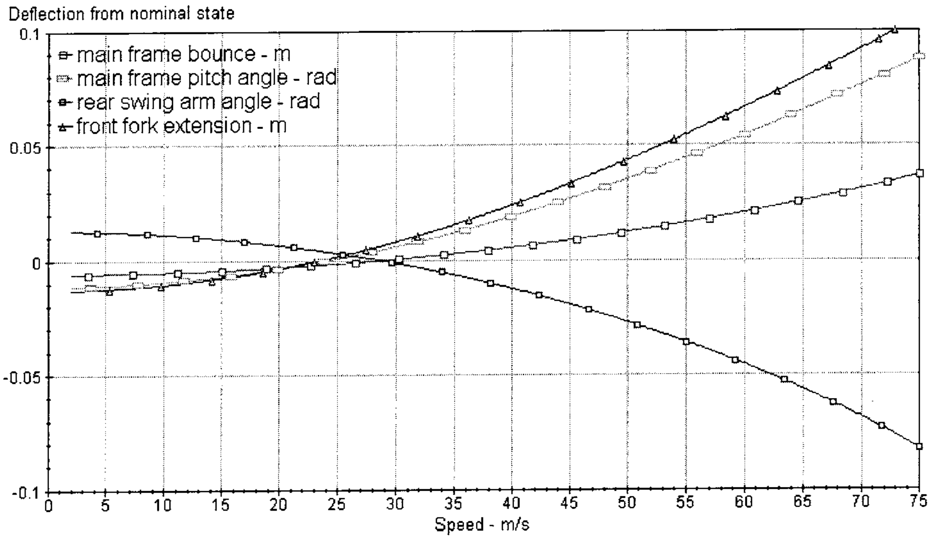

Figure 6. Quasi-equilibrium states for the straight running motorcycle decelerating at 0.2 m/s/s from high speed under the action of aerodynamic forces.

For straight running, the symmetry of the motorcycle leads to a zero state for all the out-of-plane variables corresponding to equilibrium. The aerodynamic forces cause some changes with speed in the in-plane variables and wheel loadings. Put another way, the aerodynamic forces cause a change in the machine posture with speed. The quasi-steady solutions are illustrated in Figure 6, covering deceleration from 75 to 2 m/s in 365 s. The bounce and pitch of the main frame, the angle of the swing arm and the front fork suspension displacement are shown. The resulting quasi-equilibrium states were then used by the linear analysis program to generate the root-locus plot in Figure 7.

The loci are of the form expected, showing weave and wobble modes with their known dependencies (in general terms) on speed and the in-plane modes involving bounce, pitch and wheel hop. As expected, the pitch, bounce and wheel hop modes are relatively insensitive to speed variations. Note that the nomenclature is due to Koenen. Although the loci are similar to those of Koenen, they are certainly not identical.

For the cornering cases, the general procedure is the same. The only new complexity comes from the fact that the cornering equilibrium states involve changes in the out-of-plane variables in addition to those shown in Figure 6. Some of the variables comprising the steady state solution for 0.4 rad lean angle are shown in Figure 8. The full set entails, in addition, the rider upper body lean angle, the frame twist angle, the main frame lateral velocity, the yaw velocity, the tyre side-forces from slip and camber and the tyre aligning moments. The notable features in these equilibrium state results are the accuracy with which the steering controller maintains the desired lean angle and the relatively large values of steer angle for

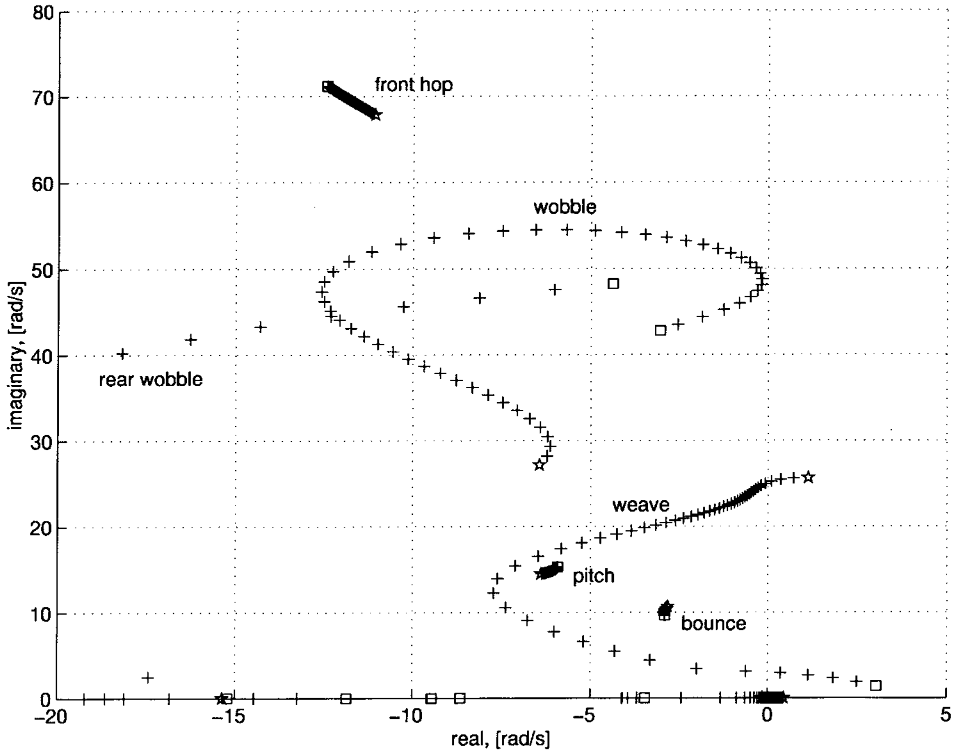

Figure 7. Root-locus plot for standard motorcycle in straight running over the speed range 2 to 75 m/s. The lowest speed points are marked with squares, while the highest speed points are marked with stars.

low speed operation. The controllers are, of course, only used as a means of solving the equilibrium equations. They are omitted in the calculation of the root loci, so that these loci relate to the uncontrolled motorcycle.

Loci for 0.4 rad lean and for 0.8 rad lean are shown in Figures 9 and 10, respectively.

At 0.4 rad lean, the weave mode at high speed is de-stabilised just a little as compared with straight running. At high speed, a slow exponential instability also occurs. The wobble mode locus is not changed significantly. Strictly, of course, these modes involve both out-of-plane and in-plane variables and a full verbal description would require more complex terminology, but we will refrain from elaborating this point. At 0.8 rad lean, a 1 to 1.5 Hz mode is marginally unstable from low to medium speeds, when it stabilises. The cornering weave mode becomes unstable at approximately the same high speed as for 0.4 rad lean angle case.

The influences of suspension damping changes on the 0.8 rad lean angle case are shown in Figs 11 and 12. The damper coefficients are doubled in the one case and set to zero in the other. For this lean angle, the suspension dampers are shown

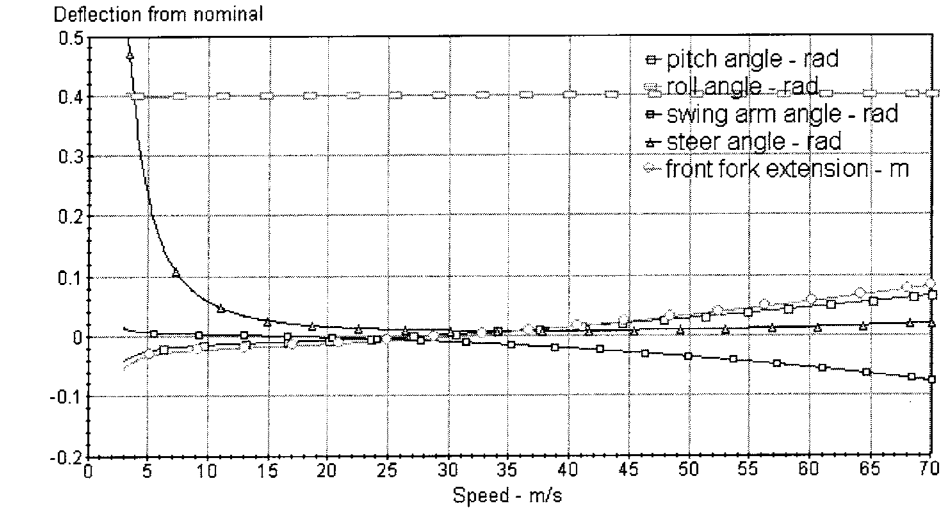

Figure 8. Equilibrium values for 0.4 rad lean angle for motorcycle decelerating at 0.23 m/s/s.

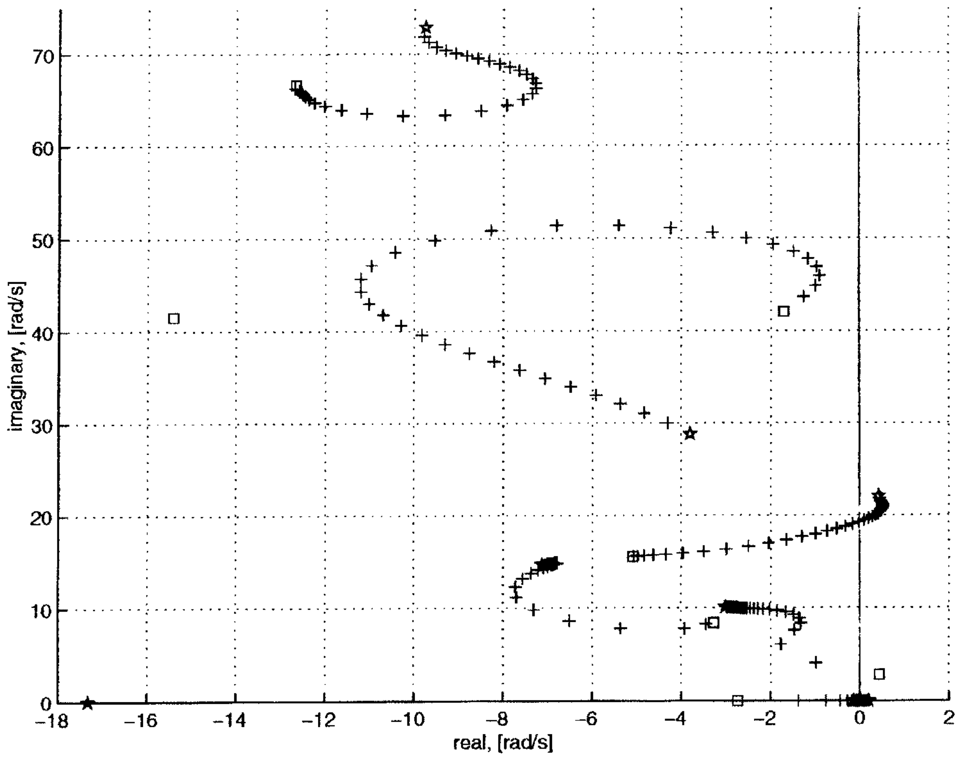

Figure 9. Root-locus plot for standard motorcycle at 0.4 rad lean angle over the speed range 6.9 m/s (squares) to 73.2 m/s (stars).

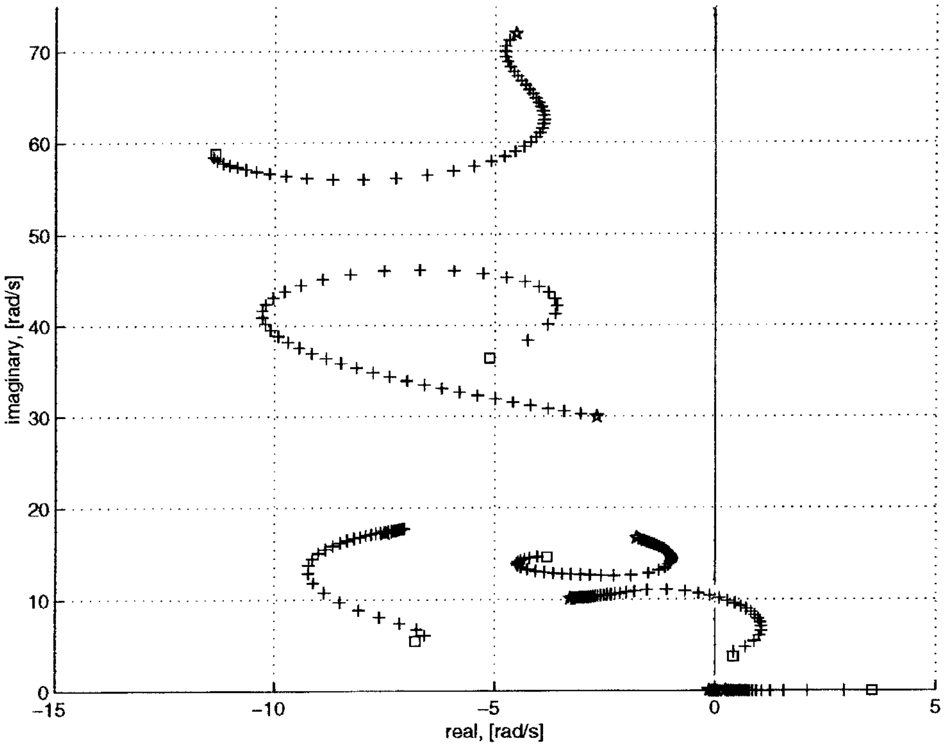

Figure 10. Root-locus plot for standard motorcycle at 0.8 rad lean angle over the speed range 7 m/s (squares) to 68 m/s (stars).

to play a role in stabilising the machine and this accords with the conventional practical wisdom. With double suspension damping, both the problem modes are substantially improved. With no suspension damping, unstable oscillations with frequencies 1 to 1.5 Hz, 3 Hz and 10 Hz all appear. This is contrary to Koenen's predictions, reinforcing the view that the present results are substantially at variance with those in Koenen's studies and much more closely aligned with practical evidence.

6. Conclusions

A significant contribution to modelling the steering behaviour of motorcycles has been made. The model is thought to be more general and more powerful than any other in the public domain. Its generality implies that it requires quite extensive parametric data for geometry, masses and mass distributions, frame and suspension stiffnesses and tyre forces and moments. The AutoSim code can be down-loaded from http://www.ee.ic.ac.uk/control/motorcycles/. It can be used directly by AutoSim users and, of course, can easily be developed by them to their own taste.

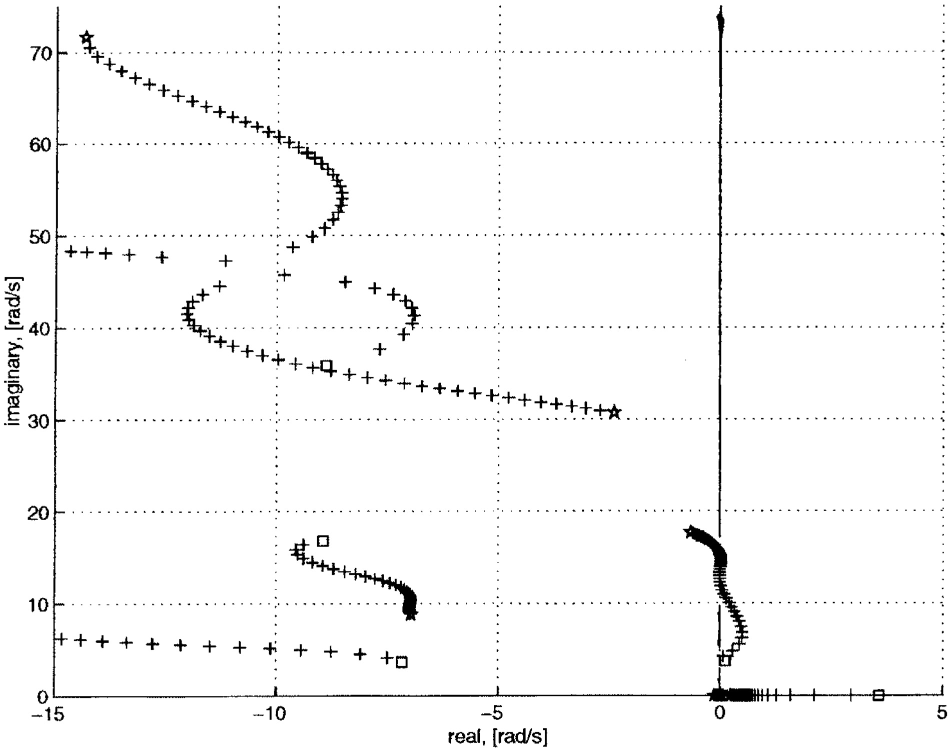

Figure 11. Root-locus plot for modified motorcycle at 0.8 rad lean angle over speed range 7 m/s (squares) to 68 m/s (stars); front and rear suspension damping coefficients twice nominal.

Alternatively, the ideas exposed can be used in other model building systems. The discussions of the geometric complexity in the steering system and front tyre contact with the ground and the diagnostic checking of the steady turning equilibrium should be particularly helpful. There is novelty also in the employment of feedback controllers to efficiently establish specified cornering equilibrium states prior to testing the stability.

The fundamental patterns of behaviour in the case of the cornering machine established by Koenen have been reproduced. In particular, the in-plane and out-of-plane modes, which are decoupled for straight running, become increasingly interdependent as the steady turn lean angle increases and root loci generated show patterns similar to those of Koenen. However, the detailed results are not the same and it must be concluded that one, or the other sets of results are erroneous. In this context, the achievement of results using only hand analysis and coding required monumental dedication by Koenen, and it should not be considered surprising that the details cannot be reproduced. The present results gain some credibility from the use of an automated model builder with a history of successes [28] and from the critical use of diagnostic checks, especially of the steady turning equilibrium states.

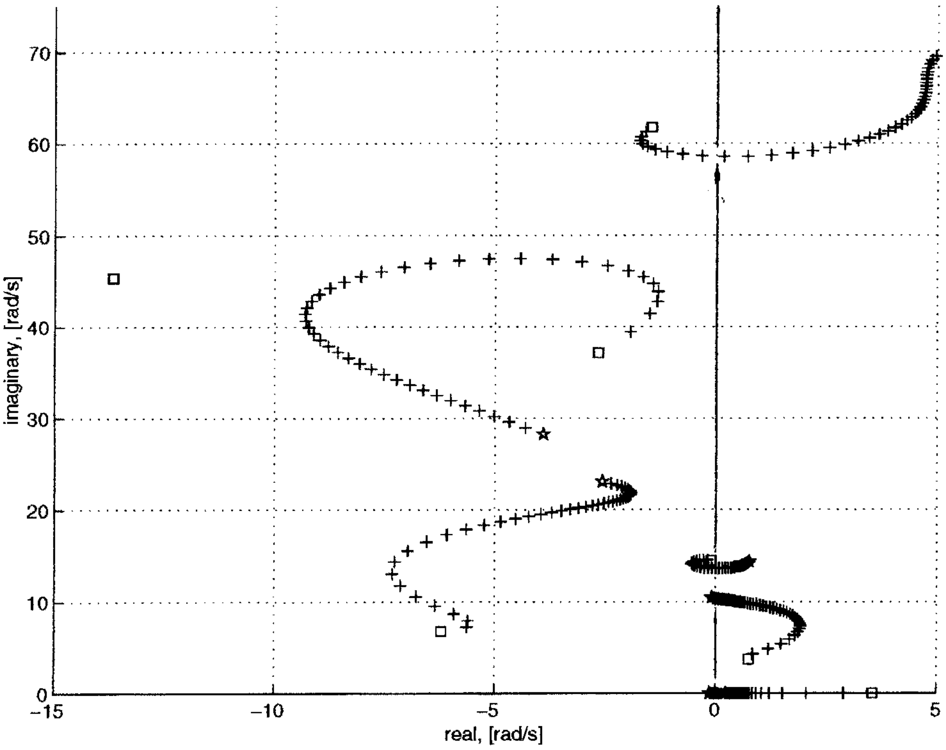

Figure 12. Root-locus plot for modified motorcycle at 0.8 rad lean angle over speed range 7 m/s (squares) to 68 m/s (stars); both front and rear suspension damping coefficients set to zero.

With respect to the perceived major prediction problem from Koenen's work, concerning the influence of suspension damping on the stability of the cornering weave mode, the present results conflict with Koenen and agree with experimental evidence [8, 13] and anecdotal evidence. It is believed that the model described is sound and that it provides a base for the predictive design of motorcycles and further analysis of motorcycle cornering behaviour.

Recent accident investigations suggest that road profile forcing has a role to play in exciting unstable oscillatory responses. As a result, road profiling is being incorporated into our models at the present time. We have a particular interest in motorcycle speed and road wavelength conditions which produce a resonant excitation of the lightly damped modes. This coincidence of circumstances has certainly been observed in relation to the cornering weave problem [29]. Further, nonlinear behaviour, giving significance to whole number relationships between forcing and response frequencies, is of great interest and is to be examined by simulation and other means in an ongoing project supported by the UK Engineering and Physical Sciences Research Council.

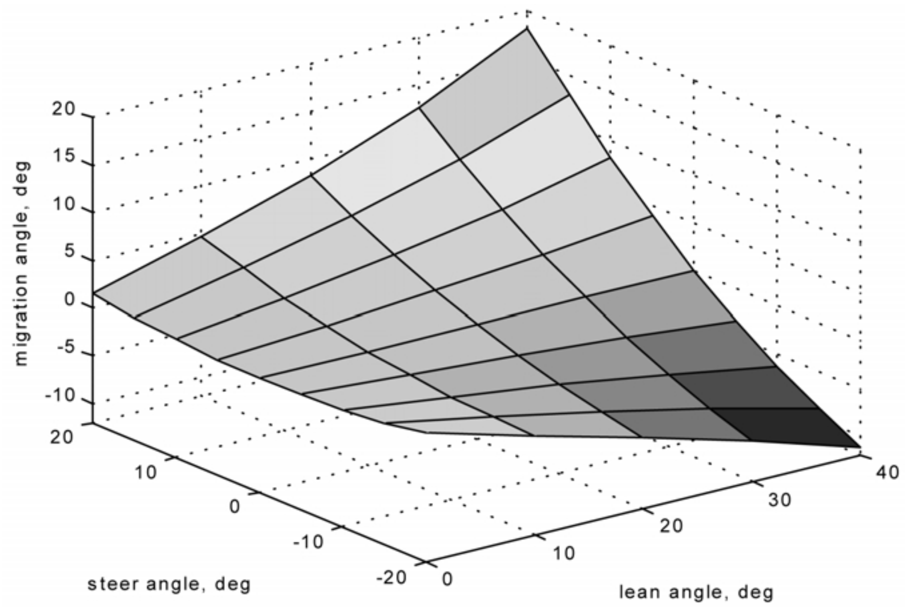

Figure A1. Front tyre contact point migration angle as a function of lean angle and steer angle, for a motorcycle with 29.8 degrees steering head angle.

Appendix: Migration of Contact Point Round Tyre Circumference for Large Lean and Steer Angles

To find the extent of the migration of the front tyre/road contact point away from its nominal position with respect to the front suspension (lower forks) body, an AutoSim kinematic analysis program was written, see http://www.ic.ac.uk/control/motorcycles/. This program treats a rotating thin disc wheel, with its axle subject to successive rotations, roll, pitch, twist (about an axis inclined to the horizontal at the steering head angle) and steer. It derives the vertical projection of the wheel radius vector as a function of the wheel spin angle, given specific values of roll, pitch, twist and steer angles. This result is contained in symbolic form in the RTF file written by AutoSim. It is of the form: $a * \cos q + b * \sin q$, where q is the wheel spin angle. Details of the expressions a and b can be seen in the MATLAB listing at http:/www.ee.ic.ac.uk/control/motorcycles/. The RTF file expressions were copied and pasted across into this code.

The condition on q for the projection of the radius vector on the vertical to be a maximum is that $\mathrm{d}/\mathrm{d}q(a * \cos q + b * \sin q) = 0$; i.e. $\tan q = b/a$. The MATLAB program computes q for given values of the angles and the results are plotted in Figure A1 for the special case of 29.8 degrees steering head angle, zero pitch and zero twist. Pitching of the bike causes the contact point to migrate to the extent of the pitch angle, in the appropriate direction, irrespective of the lean and steer angles. The frame twist angle is always very small in practice and its influence has been found, by just a few trials, to be a small fraction of the twist angle itself.

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