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Fault Isolation Filter with Linear Matrix Inequality Solution to Optimal Decoupling

Imad M. Jaimoukha, Zhenhai Li and Emmanuel Mazars

Abstract— **In this paper we consider a model–based fault detection and isolation problem for linear time–invariant dynamic systems subject to faults and disturbances. We use an observer scheme that cancels the system dynamics and defines a residual vector signal that is sensitive only to faults and disturbances. We then design a stable fault isolation filter such that the $\mathcal{H}_\infty$–norm of the transfer matrix function from disturbances to the residual is minimized (for fault detection) subject to the constraint that the transfer matrix function from faults to residual is equal to a pre–assigned diagonal transfer matrix (for fault isolation). The optimization of disturbance decoupling is accomplished via the help of linear matrix inequalities. A numerical example is also presented to illustrate the algorithm.**

I. INTRODUCTION

Model–based fault detection and isolation (FDI) schemes exploiting analytic redundancy have received increasing attention in the literature and applications [6], [18] and [11]. The schemes involve the design of an observer which effectively cancels the (nominal) process dynamics and provides a residual signal that is sensitive only to disturbances, plant/model mismatch (often recast as disturbances) and faults. The filter design objective is then to reduce the sensitivity to disturbances and/or plant/model mismatch as well as isolating faults. There are two main approaches for achieving the detection objective, namely, exact and almost disturbance decoupling [21]. In the former, the aim is to decouple the residual signal from disturbances exactly, while in the latter, the transfer matrix from disturbances to the residual signal is required to be small in either the $\mathcal{H}_2$ or $\mathcal{H}_\infty$ norm sense. For the purpose of isolating faults, in both cases, the transfer matrix function from faults to residual is required to be diagonal.

Patton and Chen considered left and right eigenvector assignment [17] and Chen *et al.* treated the robust FDI problem by using an unknown input observer with disturbances exactly decoupled in the state estimation error [2]. Although these techniques can achieve disturbance decoupling, their isolation ability is restricted. In most perfect decoupling and isolation cases, solvability conditions are generically difficult to be satisfied. Hence, almost decoupling has been widely investigated recently through $\mathcal{H}_\infty$ techniques [21]. Frank and Ding developed a matrix factorization method to obtain an optimal fault detection filter [8] and Sadrnia *et al.* utilized a Riccati equation iteration method to construct an extended $\mathcal{H}_\infty$ filter [22]. Using these techniques, the decoupling problem can be transformed to a sensitivity optimization problem, which seeks to increase the sensitivity of the residual to faults and simultaneously reduce the sensitivity to disturbances and plant/model mismatch. However, isolation is employed indirectly in the above methods through the use of banks of observers. This makes it hard to deal with multiple faults (where faults might occur simultaneously).

It is desirable to consider decoupling disturbances approximately and isolating multiple faults using a single observer. See [9] for a discussion of this issue. In this contribution, we construct an FDI observer such that the $\mathcal{H}_\infty$–norm of the transfer matrix function from disturbances to the residual is minimized, with the constraint that the transfer matrix function from faults to residual is equal to a pre–assigned diagonal transfer matrix. Necessary and sufficient condition in the square case and sufficient condition in the non-square case for the existence of such an observer are given. Various versions of this problem have been considered in the literature. The problem of designing a stable diagonalizing filter has been considered in [14] where a partial solution was given (when the number of outputs is equal to the number of faults), however, the influence of the disturbances was not considered. In [4], the problem of limiting the influence of the disturbances was considered, although the resulting equations were nonlinear and the solution was suboptimal. Here, we design a stable multiple faults isolating filter and minimize the effect of disturbances (using the $\mathcal{H}_\infty$–norm as a measure) on the residual using linear matrix inequalities (LMI). The LMI techniques used in [12] for fault detection could also be modified for fault isolation under certain assumptions. In this work, we remove these assumptions and present the work in a more general setting.

This work is organized as follows. We review the residual signal generation for almost detection and isolation filter in Sections II and III. In Section IV we divide the isolation problem into two cases and give existence conditions for their solution. The disturbance decoupling problem is treated using LMI techniques in Section V. Section VI gives a jet engine example to validate the approach. Finally, Section VII concludes the paper.

The notation we use is fairly standard. The set of real (complex) $n \times m$ matrices is denoted by $\mathcal{R}^{n\times m}$ ($\mathcal{C}^{n\times m}$). For $A \in \mathcal{R}^{n\times m}$ we use the notation A^T to denote the transpose. The set of complex numbers is denoted by $\mathcal{C}$. The open left half of the complex plane is denoted by $\mathcal{C}_-$ and the closed

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I. M. Jaimoukha, Z. Li and E. Mazars are with the Control and Power Group, Department of Electrical and Electronic Engineering, Imperial College, London SW7 2BT, UK. i.jaimouka@imperial.ac.uk zhenhai.li@imperial.ac.uk and e.mazars@imperial.ac.uk Fax: +44 207 594 6282. Telephone: +44 207 594 6279.

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right half of the complex plane is denoted by $\bar{\mathcal{C}}_+$. The i–th eigenvalue of $A \in \mathcal{C}^{n\times n}$ is denoted by $\lambda_i(A)$. For a symmetric matrix $A \in \mathcal{R}^{n\times n}$, $A > 0$ ($A < 0$) denotes that A is positive definite (negative definite), that is, $\lambda_i(A) > 0,\ \forall\, i$ ($\lambda_i(A) < 0,\ \forall\, i$). The notation $A = \operatorname{diag}(a_1, \ldots, a_n)$ denotes that A is a diagonal matrix with diagonal entries $a_1, \ldots, a_n$. The $n \times n$ identity matrix is denoted as I_n and the $n \times m$ null matrix is denoted as $0_{n,m}$ with the subscripts dropped if they can be inferred from context.

$\mathcal{R}(s)^{m\times p}$ denotes the set of all $m \times p$ proper, real–rational matrix functions of s. $\mathcal{L}_\infty^{m\times p}$ denotes the space of $m \times p$ matrix functions with entries bounded on the extended imaginary axis. The subspace $\mathcal{H}_\infty^{m\times p} \subset \mathcal{L}_\infty^{m\times p}$ denotes matrix functions analytic in the closed right–half of the complex plane. A prefix $\mathcal{R}$ denotes a real–rational function, so that $\mathcal{RH}_\infty^{m\times p}$ denotes the set of all $m \times p$ stable real–rational matrix functions of s.

For $G(s) \in \mathcal{RH}_\infty^{m\times p}$, we define

$$\|G\|_\infty = \sqrt{\sup_{\omega\in\mathcal{R}} \max_i \ \lambda_i\left(G(-j\omega)^T G(j\omega)\right)}.$$

If $G(s) \stackrel{s}{=} (A, B, C, D) \in \mathcal{RH}_\infty^{m\times p}$ with $m \geq p$, then $G(s)$ is called co–outer if $G(s)$ has no zeros [1] in $\bar{\mathcal{C}}_+$, that is, if the matrix pencil

$$\begin{bmatrix} A-\mu I & B \\ C & D \end{bmatrix}$$

has full column rank for all $\mu \in \bar{\mathcal{C}}_+$ (equivalently, if $G(s)$ has a left inverse in $\mathcal{RH}_\infty^{p\times m}$ [5]). The matrix $A \in \mathcal{R}^{n\times n}$ is called stable if $\lambda_i(A) \in \mathcal{C}_-,\ \forall\, i$. The pair (A, C) is called detectable if there exists a real matrix L such that $A + LC$ is stable. The pair (A, C) is detectable if the matrix pencil $\begin{bmatrix} A^T-\mu I & C^T \end{bmatrix}^T$ has full column rank for all $\mu \in \bar{\mathcal{C}}_+$.

II. PROBLEM FORMULATION

A faulty linear time-invariant (LTI) dynamic system subject to disturbances can be modeled as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_d d(t) + B_f f(t) + Bu(t), \\ y(t) &= Cx(t) + D_d d(t) + D_f f(t) + Du(t), \end{aligned}$$

where $x(t) \in \mathcal{R}^n$, $u(t) \in \mathcal{R}^{n_u}$ and $y(t) \in \mathcal{R}^{n_y}$ are the state, input and output vectors, respectively, and $d(t) \in \mathcal{R}^{n_d}$ and $f(t) \in \mathcal{R}^{n_f}$ are the disturbance and fault vectors, respectively. Here, $B_f \in \mathcal{R}^{n\times n_f}$ and $D_f \in \mathcal{R}^{n_y\times n_f}$ are the component and instrument fault distribution matrices, respectively, while $B_d \in \mathcal{R}^{n\times n_d}$ and $D_d \in \mathcal{R}^{n_y\times n_d}$ are the corresponding disturbance distribution matrices [7], [6]. For simplicity, structured model uncertainties are recast as additive disturbances [17]. We assume that the system has at least as many outputs as potential faults, i.e, $n_y \geq n_f$, which is a general assumption in fault diagnosis. Since sensor faults can be recast by actuator faults [3], [15], [16], [24], we may also assume, without loss of generality, that $D_f = 0$.

A standard filter–based FDI approach to generate a residual signal $r(t)$ is the use of system duplication to cancel the the input dynamics [6]. Define

$$\begin{bmatrix} G_d(s) & G_f(s) \end{bmatrix} \stackrel{s}{=} \left[\begin{array}{c|cc} A & B_d & B_f \\ \hline C & D_d & 0 \end{array}\right] \in \mathcal{R}(s)^{n_y\times(n_d+n_f)}.$$

Then the residual dynamics are

$$r(s) = F(s)[G_d(s)d(s) + G_f(s)f(s)], \tag{1}$$

where $F(s) \in \mathcal{R}(s)^{n_f\times n_y}$ is a free stable post-filter to be designed.

The residual dynamics can be generated using a state–observer framework as

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) - L\left(y(t) - C\hat{x}(t) - Du(t)\right), \\ r(t) &= H(y(t) - C\hat{x}(t) - Du(t)), \end{aligned}$$

where $\hat{x}(t) \in \mathcal{R}^n$ is the observer state and $r(t) \in \mathcal{R}^{n_f}$ is the residual signal. Here $L \in \mathcal{R}^{n\times n_y}$ and $H \in \mathcal{R}^{n_f\times n_y}$ are the observer and residual gain matrices, respectively, and are to be determined. Define the state estimation error signal as $e(t) = x(t) - \hat{x}(t)$. It follows that the residual dynamics are given by

$$\begin{aligned} \dot{e}(t) &= (A + LC)e(t) + (B_d + LD_d)d(t) + B_f f(t), \\ r(t) &= HCe(t) + HD_d d(t). \end{aligned}$$

Taking Laplace transforms, $r(s) = T_{rf}(s)f(s) + T_{rd}(s)d(s)$, where

$$\begin{bmatrix} T_{rd}(s) & T_{rf}(s) \end{bmatrix} \stackrel{s}{=} \left[\begin{array}{c|cc} A+LC & B_d+LD_d & B_f \\ \hline HC & HD_d & 0 \end{array}\right], \tag{2}$$

are the transfer matrices from faults and disturbances to residuals, respectively. Then a calculation shows that $r(s) = F(s)[G_d(s)d(s) + G_f(s)f(s)]$, where

$$F(s) \stackrel{s}{=} \left[\begin{array}{c|c} A + LC & L \\ \hline HC & H \end{array}\right] \in \mathcal{R}(s)^{n_f\times n_y}, \tag{3}$$

so that

$$\begin{bmatrix} T_{rd}(s) & T_{rf}(s) \end{bmatrix} = F(s)\begin{bmatrix} G_d(s) & G_f(s) \end{bmatrix}. \tag{4}$$

To guarantee the existence of at least one L such that $A + LC$ is stable, we assume that the pair (A, C) is detectable. In fact, with this assumption, we show next that there is no loss of generality in assuming that A is stable. If A is unstable and (A, C) is detectable, there exists L_0 such that $A + L_0C$ is stable. A simple calculation now shows that

$$\begin{bmatrix} T_{rd}(s) & T_{rf}(s) \end{bmatrix} = \bar{F}(s)\begin{bmatrix} \bar{G}_d(s) & \bar{G}_f(s) \end{bmatrix}$$

where

$$\begin{bmatrix} \bar{G}_d(s) & \bar{G}_f(s) \end{bmatrix} \stackrel{s}{=} \left[\begin{array}{c|cc} A + L_0C & B_d + L_0D_d & B_f \\ \hline C & D_d & 0 \end{array}\right]$$

and where

$$\bar{F}(s) \stackrel{s}{=} \left[\begin{array}{c|c} A + L_0C + (L - L_0)C & L - L_0 \\ \hline HC & H \end{array}\right].$$

The result now follows by identifying $A := A + L_0C$, $B_d := B_d + L_0D_d$ and $L := L - L_0$. Note, however, that fault detection and isolation for unstable systems may be problematic in practice due to modeling uncertainties [13].

We consider the following detection and isolation problem.

Problem 2.1: Find a stable filter $F(s)$ such that the optimum

$$\gamma_o = \inf_{\substack{T_{rf}=T_o \\ F(s)\in\mathcal{H}_\infty^{n_f\times n_y}}} \|T_{rd}\|_\infty \quad (5)$$

is achieved, where

$$T_o = M(sI-\Lambda)^{-1} \quad (6)$$

is a preassigned stable diagonal transfer matrix and

$$\begin{aligned} \Lambda &= \mathrm{diag}(\lambda_1,\ldots,\lambda_{n_f}) \in \mathcal{R}^{n_f\times n_f}, \quad \lambda_i<0,\ \forall i, \\ M &= \mathrm{diag}(m_1,\ldots,m_{n_f}) \in \mathcal{R}^{n_f\times n_f}, \quad |m_i|>0,\ \forall i. \end{aligned} \quad (7)$$

It is evident that the first constraint $T_{rf} = FG_f = T_o$ is equivalent to $F\hat{G}_f = I$, where

$$\begin{aligned} \hat{G}_f &= G_f(sI-\Lambda)M^{-1} = sG_fM^{-1} - G_f\Lambda M^{-1} \\ &\stackrel{s}{=} \left[\begin{array}{c|c} A & AB_fM^{-1} \\ \hline C & CB_fM^{-1}\end{array}\right] - \left[\begin{array}{c|c} A & -B_f\Lambda M^{-1} \\ \hline C & 0\end{array}\right] \\ &\stackrel{s}{=} \left[\begin{array}{c|c} A & AB_fM^{-1}-B_f\Lambda M^{-1} \\ \hline C & CB_fM^{-1}\end{array}\right]. \end{aligned} \quad (8)$$

Hence, Problem 2.1 is equivalent to the following

Problem 2.2: Find a stable filter $F(s)$ such that the optimum

$$\gamma_o = \inf_{\substack{F\hat{G}_f=I \\ F(s)\in\mathcal{H}_\infty^{n_f\times n_y}}} \|FG_d\|_\infty \quad (9)$$

is achieved.

Remark 2.1: The equality constraint $F\hat{G}_f = I$ follows from the isolation condition, which constructs a directional residual to achieve fault isolation. Correspondingly, the minimization of T_{rd} in the sense of $\mathcal{H}_\infty$-norm is interpreted as the detection or disturbance decoupling condition.

Remark 2.2: Problem 2.2 is known as an almost disturbance decoupling problem in fault diagnosis [21], where disturbance signatures do not need to be perfectly removed from the residual. It is also classified as an optimal approximate decoupling problem in [9], where fault response are specified as explicit equality constraints instead of inequalities constraints. This setup allows an easy implementation of isolation functions.

III. STABILITY CONDITION

Before the design of a residual generator, we give the existence condition of a stable isolation filter first.

Lemma 3.1: Suppose $n_y \geq n_f$. Then the following are equivalent:

1) There exists a stable fault isolation filter $F(s)$.
2) $\hat{G}_f$ is co–outer.
3) CB_f has full column rank and $G_f(s)$ has no finite zeros in $\bar{\mathcal{C}}_+$.

Proof.

$1\to2$ The result follows from the fact that $\hat{G}_f$ is stable (from (8) and the assumption that A stable), and the constraint in (9):

$$F\hat{G}_f = I. \quad (10)$$

$2\to1$ Follows directly from the definition of co–outer functions.

$2\leftrightarrow3$ Let $\rho(\mu)$ be defined as

$$\rho(\mu) := \mathrm{rank}\left(\begin{bmatrix} A-\mu I & (AB_f-B_f\Lambda)M^{-1} \\ C & CB_fM^{-1}\end{bmatrix}\right) \quad (11)$$

For any finite $\mu\in\bar{\mathcal{C}}_+$, $ME := M(\mu I-\Lambda)^{-1}$ has full rank since Λ is stable and so

$$\begin{aligned} \rho(\mu) &= \mathrm{rank}\left(\begin{bmatrix} A-\mu I & (AB_f-B_f\Lambda)M^{-1} \\ C & CB_fM^{-1}\end{bmatrix}\begin{bmatrix} I & -B_fE \\ 0 & ME\end{bmatrix}\right) \\ &= \mathrm{rank}\left(\begin{bmatrix} A-\mu I & B_f \\ C & 0\end{bmatrix}\right). \end{aligned}$$

It follows that $\hat{G}_f$ and $G_f(s)$ have the same finite zeros in $\bar{\mathcal{C}}_+$ and the conclusion follows. Note the fact that, due to the nonsingularity of M, CB_f having full column rank is equivalent to CB_fM^{-1} having full column rank. □

Remark 3.1: Part (3) in Lemma 3.1 gives a test independent of Λ and could be used in the beginning of the design.

Remark 3.2: The condition that CB_f has full column rank seems to be restrictive, however, it is necessary for the form of T_o we have chosen (e.g., diagonal with first order diagonal elements). Other choices of T_o will require different corresponding conditions. In this work we have opted for the simplest choice of $T_o(s)$, e.g., fixed, diagonal and with first order diagonal elements. An issue for further research is to optimize the choice of $T_o(s)$.

Remark 3.3: In the case that $n_y > n_f$, then, generically, $\hat{G}_f$ has no zeros and is co–outer. Thus when the number of outputs is larger than the number of faults, we expect the fault isolation problem to be solvable.

IV. FAULT ISOLATION FILTER DESIGN

Next, we derive an optimal fault isolation filter for two cases: the square case ($n_y = n_f$) and the general case ($n_y > n_f$). We will show that, the result in the square case can be generalized to the general case via solving a left inverse of a transfer matrix in state space.

Theorem 4.1: Suppose $n_y = n_f$. Then there exists an optimal filter $F(s)$ which solves Problem 2.2 if and only if $\hat{G}_f$ is an outer function. Furthermore, if a feasible $F(s)$ exists, then the corresponding observer gains in the form of (3) are given as

$$L = -(AB_f - B_f\Lambda)(CB_f)^{-1}, \quad (12)$$

$$H = M(CB_f)^{-1}. \quad (13)$$

Proof. Firstly, we show that the optimization objective in (9) can be achieved if and only if $F = \hat{G}_f^{-1}$ exists and stable.

($\Rightarrow$) According to Lemma 3.1, the existence of a feasible F implies the existence of a stable $\hat{G}_f^{-1}$.

($\Leftarrow$) If $\hat{G}_f^{-1}$ is stable, then (9) is equivalent to

$$\begin{aligned} \gamma_o &= \inf_{\substack{F\hat{G}_f=I \\ F(s)\in\mathcal{H}_\infty^{n_f\times n_y}}} \left\|F\hat{G}_f\hat{G}_f^{-1}G_d\right\|_\infty \\ &= \inf \left\|\hat{G}_f^{-1}G_d\right\|_\infty, \end{aligned}$$

where the optimum is achieved when $F = \hat{G}_f^{-1}$.

Secondly, the selection of L and H can be verified via comparing (3) with

$$\begin{aligned} F &= \hat{G}_f^{-1} \\ &\stackrel{s}{=} \left[\begin{array}{c|c} A - R_F M^{-1} S_F C & -R_F M^{-1} S_F \\ \hline M(CB_f)^{-1} C & M(CB_f)^{-1} \end{array}\right] \\ &\stackrel{s}{=} \left[\begin{array}{c|c} A - R_F (CB_f)^{-1} C & -R_F (CB_f)^{-1} \\ \hline M(CB_f)^{-1} C & M(CB_f)^{-1} \end{array}\right], \end{aligned}$$

where $R_F = AB_f - B_f\Lambda$ and $S_F = (CB_f M^{-1})^{-1}$.

Finally, an outer $\hat{G}_f$ implies that $\hat{G}_f$ has full rank over the extended imaginary axis and that $\hat{G}_f)^{-1}$ is stable, which in turn imply that CB_f is nonsingular and $A - (AB_f - B_f\Lambda)(CB_f)^{-1}C$ is stable. □

In the square case, the almost isolation condition can be easily verified by calculating the inverse of $\hat{G}_f$. Note, however, that the FI filter is unique and no degrees of freedom can be used to reduce the effects of disturbances. It is natural to generalize the result to the non–square case ($n_y > n_f$). However, the left-inverse of $\hat{G}_f$ is not unique and degrees of freedom may be exploited in an observer-based form. Hence, in the rest of this section, we consider a state-space solution of $F\hat{G}_f = I$, in which the freedom in the choice of the solutions for L and H will be used to optimize disturbance decoupling.

Lemma 4.1: Suppose that $n_y > n_f$. Then the isolation condition (10) is satisfied if there exist L and H such that

$$\begin{aligned} LCB_f &= B_f\Lambda - AB_f \\ HCB_f &= M. \end{aligned} \qquad (14)$$

Furthermore, with the assumption $\hat{G}_f$ is co-outer, the feasible L and H are given by

$$L = L_1 + RL_2 \qquad (15)$$

$$H = H_1 + SL_2, \qquad (16)$$

where $L_1 = (B_f\Lambda - AB_f)D^\#$, $L_2 = I - DD^\#$, $H_1 = MD^\#$ and $D^\# = (D^TD)^{-1}D^T$ is the Moore–Penrose generalized inverse of $D = CB_f$. In addition, $R \in \mathcal{R}^{n\times n_y}$ and $S \in \mathcal{R}^{n_f\times n_y}$ are free matrices to be decided.

Proof. By substituting (3) and (8) into (10), we can show that

$$\begin{aligned} F\hat{G}_f &\stackrel{s}{=} \left[\begin{array}{c|c} A+LC & L \\ \hline HC & H \end{array}\right] \left[\begin{array}{c|c} A & AB_fM^{-1} - B_f\Lambda M^{-1} \\ \hline C & CB_fM^{-1} \end{array}\right] \\ &\stackrel{s}{=} \left[\begin{array}{cc|c} A+LC & LC & LCB_fM^{-1} \\ 0 & A & AB_fM^{-1} - B_f\Lambda M^{-1} \\ \hline HC & HC & HCB_fM^{-1} \end{array}\right] \\ &\stackrel{s}{=} \left[\begin{array}{c|c} T^{-1}\begin{bmatrix} A+LC & LC \\ 0 & A \end{bmatrix} T & T^{-1}\begin{bmatrix} LCB_fM^{-1} \\ AB_fM^{-1} - B_f\Lambda M^{-1} \end{bmatrix} \\ \hline \begin{bmatrix} HC & HC \end{bmatrix} T & HCB_fM^{-1} \end{array}\right] \\ &\stackrel{s}{=} \left[\begin{array}{cc|c} A+LC & 0 & LCB_fM^{-1} + AB_fM^{-1} - B_f\Lambda M^{-1} \\ 0 & A & AB_fM^{-1} - B_f\Lambda M^{-1} \\ \hline HC & 0 & HCB_fM^{-1} \end{array}\right] \\ &\stackrel{s}{=} \left[\begin{array}{c|c} A+LC & LCB_fM^{-1} + AB_fM^{-1} - B_f\Lambda M^{-1} \\ \hline HC & HCB_fM^{-1} \end{array}\right] = I, \end{aligned}$$

where the similarity transformation matrix $T = \begin{bmatrix} I & -I \\ 0 & I \end{bmatrix}$. Hence, (14) gives a sufficient condition to ensure that (10) is satisfied.

According to Lemma 3.1, an co-outer $\hat{G}_f$ implies that CB_f has full column rank. Therefore, Moore–Penrose generalized inverse can be applied to solve (14), which results in (15) and (16). □

V. AN LMI SOLUTION TO OPTIMAL DISTURBANCE DECOUPLING

Linear matrix inequality techniques have proved to be popular in control system analysis and design due to their numerical reliability. The applications of LMI in model-based FD have been addressed in [10], [19] and [25], where LMI techniques are used to solve a sensitivity optimization problem, however, isolation ability was not emphasized.

In this section, we use the freedom provide in Lemma 4.1 to minimize residual sensitivity to disturbances, which is achieved using the Bounded Real Lemma [23].

Lemma 5.1: Let $n_y > n_f$. There exist L and H such that $A + LC$ is stable and $\|T_{rd}\|_\infty < \gamma$ if and only if there exist L, H and $P = P^T \in \mathcal{R}^{n\times n}$ such that $P > 0$ and

$$\begin{bmatrix} (A+LC)^TP + P(A+LC) & P(B_d+LD_d) & C^TH^T \\ (B_d+LD_d)^TP & -\gamma I & D_d^TH^T \\ HC & HD_d & -\gamma I \end{bmatrix} < 0. \qquad (17)$$

The Bounded Real Lemma gives the solvability conditions in the form of a Bilinear Matrix Inequality. The next theorem provides a numerical algorithm to compute L and H via solving an LMI. Note that the condition is only sufficient since it is based on Lemma 4.1 which gives sufficient conditions on L and H to satisfy the isolation condition (10).

Theorem 5.1: Suppose that $n_y > n_f$ and $\hat{G}_f$ is co–outer. There exist L and H such that the specifications in Problem 2.2 are satisfied if there exist $Z \in \mathcal{R}^{n\times n_y}$, $S \in \mathcal{R}^{n_f\times n_y}$ and $P = P^T \in \mathcal{R}^{n\times n}$ such that

$$P > 0, \qquad (18)$$

and

$$\begin{bmatrix} P(A+L_1C) + ZL_2C + (\star) & (\star) & (\star) \\ (B_d+L_1D_d)^TP + D_d^TL_2^TZ^T & -\gamma I & (\star) \\ H_1C + SL_2C & H_1D_d + SL_2D_d & -\gamma I \end{bmatrix} < 0, \qquad (19)$$

where $(\star)$ denotes terms readily inferred from symmetry. Furthermore, if (18) and (19) are solved, we can construct L and H as

$$L = L_1 + P^{-1}ZL_2, \qquad (20)$$

$$H = H_1 + SL_2. \qquad (21)$$

Proof. By substituting (15) and (16) into (17) and applying Lemma 5.1, we get (18) and (19). Then (9) is achieved if the LMIs have a feasible solution. The corresponding filter is obtained by extracting $R = P^{-1}Z$ and S from the LMIs. □

Remark 5.1: Here, we design a stable fault isolating filter for the general case and, furthermore, derive an LMI algorithm for minimizing the effect of disturbances (using the $\mathcal{H}_\infty$–norm as a measure) on the residual. The results obtained generalise those of [14] and [4].

VI. NUMERICAL EXAMPLE

To illustrate the application of the isolation filter scheme in a real system, a jet engine example is considered in this section. The GE-21 jet engine state-space model [20] is given as

$$A=\begin{bmatrix} -3.370 & 1.636 \\ -0.325 & -1.896 \end{bmatrix},$$
$$B=\begin{bmatrix} 0.586 & -1.419 & 1.252 \\ 0.410 & 1.118 & 0.139 \end{bmatrix},$$
$$C=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0.731 & 0.786 \end{bmatrix},$$
$$D=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.267 & -0.025 & -0.146 \end{bmatrix}.$$

The system has $n = 2$, $n_y = 3$, and $n_u = 3$. To consider the general case ($n_y > n_f$), we suppose that this system is subject to two potential actuator faults. Here, the setup is given by

$$B_f = \begin{bmatrix} 0.586 & -1.419 \\ 0.410 & 1.118 \end{bmatrix},$$

and the disturbance distribution matrices are given as

$$B_d = \begin{bmatrix} -0.2163 & 0.0627 \\ -0.8328 & 0.1438 \end{bmatrix},$$
$$D_d = \begin{bmatrix} -0.5732 & -0.0188 \\ 0.5955 & 0.1636 \\ 0.5946 & 0.0873 \end{bmatrix}.$$

It is easy to see that $\hat{G}_f$ is a co-outer function via checking condition (3) in Lemma 3.1. A simple selection of the diagonal matrix is

$$\Lambda = diag\{-1, -2\},\ M = diag\{1, 1\},$$

which means that we allocate the same priority to each fault.

Following Lemma 4.1, we have

$$L_1=\begin{bmatrix} 1.6853 & -1.1942 & 0.2933 \\ 0.4251 & 0.0755 & 0.3701 \end{bmatrix},$$
$$L_2=\begin{bmatrix} 0.2483 & 0.2670 & -0.3397 \\ 0.2670 & 0.2871 & -0.3652 \\ -0.3397 & -0.3652 & 0.4647 \end{bmatrix},$$
$$H_1=\begin{bmatrix} 0.3732 & 0.5766 & 0.7260 \\ -0.3756 & 0.4262 & 0.0604 \end{bmatrix}.$$

Then, substituting into (19) results in the observer gains

$$L=\begin{bmatrix} -0.6025 & -3.6542 & 3.4230 \\ -0.6718 & -1.1039 & 1.8705 \end{bmatrix},$$
$$H=\begin{bmatrix} 1.1128 & 1.3719 & -0.2859 \\ 0.2938 & 1.1461 & -0.8554 \end{bmatrix}.$$

The corresponding optimal $\gamma_o = 0.2296$.

Assume that disturbance 1 is a white noise with mean zero and standard deviation 1 and disturbance 2 is a constant bias of amplitude 1 applied from the 2nd second. Fault 1 in actuator 1, simulated by an abrupt jump, and fault 2 in actuator 2, simulated by an incipient drift with slope 0.5, are connected from the 4th second and 6th second, respectively. Figure 1 gives the residual responses. The example makes clear that the designed filter satisfies the performance requirement of rapid fault detection and isolation which is sufficiently robust against disturbances.

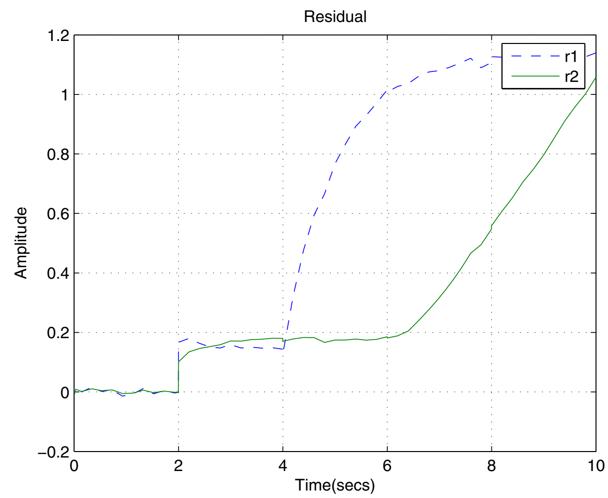

Fig. 1. Time response of the residual vector

VII. SUMMARY

We have presented the solution of a fault detection and isolation problem using a state observer framework. The fault isolation is accomplished by solving two equality constraints (see Lemma 4.1). It was also shown that the optimal FDI filter design reduces to the solution of linear matrix inequalities (see Theorem 5.1). Our scheme can handle multiple faults (where faults might occur at the same time) as well as provide robustness to disturbances. A jet engine example is given to clarify our algorithm.

VIII. ACKNOWLEDGMENTS

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