An LMI Approach to the Robust Fault Detection and Isolation Problem

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Abstract— **This paper considers the robust fault detection and isolation (FDI) problem for uncertain linear time-invariant (LTI) systems. An FDI filter minimizes the sensitivity of the residual signal to disturbances and modeling errors subject to the constraint that the transfer matrix function from the faults to the residual is close to a diagonal matrix (for fault isolation). A solution of the optimization problem is presented via the formulation of linear matrix inequalities (LMI). A jet engine example is employed to demonstrate the effectiveness of our results.**

Keywords— **Fault detection and isolation, linear matrix inequalities, robust and $\mathcal{H}_\infty$ control, multiobjective optimization.**

I. Introduction

In recent years, Model–based fault detection and isolation (FDI) schemes exploiting analytic redundancy have received increasing attention in the literature and applications [3], [5] and [7]. One of the key issues related to observer–based fault detection and isolation (FDI) systems is concerned with their robustness. Indeed, it is often the nature of industrial systems that the effects of the possible faults and disturbances are coupled and that modeling errors are unavoidable. The schemes involve the design of an observer which provides a residual signal that is sensitive only to disturbances, plant/model mismatch and faults. The filter design objective is then to reduce the sensitivity to disturbances and plant/model mismatch as well as isolating faults. The performance of an FDI system should therefore be measured by a suitable trade off between robustness and sensitivity.

Several methods for achieving robust FDI were developed such as Patton and Chen [6] who explored approaches using eigenstructure assignments. Another approach consists in constructing an unknown input observer to decouple disturbances in the phase of state estimation [1]. Frank and Ding developed a matrix factorization method to obtain an optimal fault detection filter [2] and Sadrnia *et al.* utilized a Riccati equation iteration method to construct an extended $\mathcal{H}_\infty$ filter [9]. Recently developed linear matrix inequality (LMI) approaches offer numerically attractive techniques for formulating robust FDI decoupling problems. Hou and Patton gave a realization of fault detection observer design based on the bounded real lemma [4]. Zhong *et al.* proposed a new performance index by introducing a reference residual model, formulated using LMI techniques [12]. Using these techniques, the decoupling problem can be transformed to a sensitivity optimization problem, which seeks to increase the sensitivity of the residual to faults and simultaneously reduce the sensitivity to disturbances and plant/model mismatch. However, isolation is employed indirectly in the above methods through the use of banks of observers. This makes it hard to deal with multiple faults (where faults might occur simultaneously). Furthermore, these approaches recast model uncertainties as system disturbances, which restricts the class of model uncertainties that can be handled.

In this paper, an FDI filter is constructed such that the $\mathcal{H}_\infty$–norm of the transfer matrix function from both disturbances and plant/model mismatch to the residual is smaller than a given value, with the constraint that the transfer matrix function from faults to the residual is close to a diagonal matrix. The filter can provide robustness to uncertainties in all system matrices. Sufficient conditions for the existence of such a filter are obtained in the form of LMIs.

The structure of the work is as follows. After defining the notation, we review observer–based fault isolation techniques for residual signal generation and give the problem formulation in Section II. Section III presents a matrix inequality solution to the problem. Section IV presents a solution via solving an LMI. Finally, a numerical example is presented in Section V and Section VI summarizes our results.

The notation we use is fairly standard. The set of real (complex) $n \times m$ matrices is denoted by $\mathcal{R}^{n\times m}$ ($\mathcal{C}^{n\times m}$). For $A \in \mathcal{C}^{n\times m}$ we use the notation A' to denote the complex conjugate transpose. A matrix $A \in \mathcal{C}^{n\times n}$ is called Hermitian if $A = A'$. For a Hermitian matrix $A \in \mathcal{C}^{n\times n}$, $A \geq 0$ denotes that A is positive semidefinite (that is, all the eigenvalues of A are greater than or equal to zero). For a Hermitian matrix $A \in \mathcal{C}^{n\times n}$, $\bar{\lambda}(A)$ denotes the largest and $\underline{\lambda}(A)$ the smallest eigenvalue of A, respectively. For $A \in \mathcal{C}^{n\times m}$, $\bar{\sigma}(A)$ denotes the largest, and $\underline{\sigma}(A)$ the smallest, singular values of A, respectively. Note that $\bar{\sigma}(A) = \sqrt{\bar{\lambda}(A'A)}$ and $\underline{\sigma}(A) = \sqrt{\underline{\lambda}(A'A)}$. The $n \times n$ identity matrix is denoted as I_n and the $n\times m$ null matrix is denoted as $0_{n,m}$ with the subscripts occasionally dropped if they can be inferred from context.

$\mathcal{R}(s)^{m\times p}$ denotes the set of all $m \times p$ proper, real–rational matrix functions of s. $\mathcal{L}_\infty^{m\times p}$ denotes the space of $m \times p$

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matrix functions with entries bounded on the extended imaginary axis. The subspace $\mathcal{H}_\infty^{m\times p} \subset \mathcal{L}_\infty^{m\times p}$ denotes matrix functions analytic in the closed right–half of the complex plane. A prefix $\mathcal{R}$ denotes a real–rational function, so that $\mathcal{RH}_\infty^{m\times p}$ denotes the set of all $m \times p$ stable real–rational matrix functions of s.

For $G(s) \in \mathcal{RH}_\infty^{m\times p}$ we define

$$\|G\|_\infty = \sup_{\omega\in\mathcal{R}} \bar{\sigma}\left(G(j\omega)\right) \text{ and } \|G\|_- = \inf_{\omega\in\mathcal{R}} \underline{\sigma}\left(G(j\omega)\right).$$

In Section III, we use the following version of the bounded real lemma.

Lemma 1.1: (Scherer [10], Scherer *et al*[11]). Let $G(s) \stackrel{s}{=} (A,B,C,D)$ where $A \in \mathcal{R}^{n\times n}$ and let $\gamma > 0$. Then A is stable and $\|G\|_\infty < \gamma$ if and only if there exists $P = P' \in \mathcal{R}^{n\times n}$ such that $P > 0$ and

$$\begin{bmatrix} A'P + PA & PB & C' \\ B'P & -\gamma I & D' \\ C & D & -\gamma I \end{bmatrix} < 0.$$

II. Problem formulation

Consider a linear time invariant (LTI) dynamic system subject to disturbances, modeling errors and process, sensor and actuator faults modeled as

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & (A+\Delta A)x(t) + (B+\Delta B)u(t) \\ & & +B_d d(t) + B_f f(t), \\ y(t) & = & (C+\Delta C)x(t) + (D+\Delta D)u(t) \\ & & +D_d d(t) + D_f f(t), \end{array} \right. \tag{1}$$

where $x(t) \in \mathcal{R}^n$, $u(t) \in \mathcal{R}^{n_u}$ and $y(t) \in \mathcal{R}^{n_y}$ are the process state, input and output vectors, respectively, and where $d(t) \in \mathcal{R}^{n_d}$ and $f(t) \in \mathcal{R}^{n_f}$ are the disturbance and fault vectors, respectively. Here, $B_f \in \mathcal{R}^{n\times n_f}$ and $D_f \in \mathcal{R}^{n_y\times n_f}$ are the component and instrument fault distribution matrices, respectively, while $B_d \in \mathcal{R}^{n\times n_d}$ and $D_d \in \mathcal{R}^{n_y\times n_d}$ are the corresponding disturbance distribution matrices [3]. The matrices $\Delta A, \Delta B, \Delta C, \Delta D$ represent modeling errors and are given by:

$$\begin{bmatrix} \Delta A & \Delta B \\ \Delta C & \Delta D \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \Delta \begin{bmatrix} E_1 & E_2 \end{bmatrix}, \tag{2}$$

where F_1, F_2, E_1, E_2 are known matrices and Δ is an unknown matrix such that $\Delta\Delta' \le I$. Note that this is a larger class of uncertainties than that considered in [12].

In general, a residual signal in an FDI system should represent the inconsistency between the actual system variables and the mathematical model, and respond to faults, disturbances and modeling errors only.

The objective is to design a fault detection and isolation observer (or filter) of the form

$$\left\{ \begin{array}{rcl} \dot{\hat{x}}(t) & = & A\hat{x}(t) + (B+LD)u(t) - L\left(y(t) - C\hat{x}(t)\right), \\ r(t) & = & H(y(t) - C\hat{x}(t) - Du(t)), \end{array} \right.$$

where $\hat{x}(t) \in \mathcal{R}^n$ is the observer state and $r(t) \in \mathcal{R}^{n_f}$ is the residual signal. Here $L \in \mathcal{R}^{n\times n_y}$ and $H \in \mathcal{R}^{n_f\times n_y}$ are the observer and residual gain matrices, respectively, and are to be determined. Define the state estimation error signal as $e(t) = x(t) - \hat{x}(t)$. It follows that the residual dynamics are given by

$$\left\{ \begin{array}{rcl} \dot{e}(t) & = & (A+LC)e(t) + (\Delta A + L\Delta C)x(t) \\ & & +(B_d + LD_d)d(t) + (B_f + LD_f)f(t) \\ & & +(\Delta B + L\Delta D)u(t), \\ r(t) & = & HCe(t) + H\Delta Cx(t) + H\Delta Du(t) \\ & & +HD_d d(t) + HD_f f(t). \end{array} \right.$$

By setting $z(t) = \begin{bmatrix} e(t) \\ x(t) \end{bmatrix}$ and $w(t) = \begin{bmatrix} d(t) \\ u(t) \end{bmatrix}$ we get:

$$\left\{ \begin{array}{rcl} \dot{z}(t) & = & \tilde{A}z(t) + \tilde{B}_f f(t) + \tilde{B}_w w(t), \\ r(t) & = & \tilde{H}z(t) + HD_f f(t) + \tilde{H}_w w(t), \end{array} \right. \tag{3}$$

where

$$\tilde{A} = \begin{bmatrix} A+LC & \Delta A + L\Delta C \\ 0 & A+\Delta A \end{bmatrix}, \ \tilde{H} = \begin{bmatrix} HC & H\Delta C \end{bmatrix},$$

$$\tilde{H}_w = \begin{bmatrix} HD_d & H\Delta D \end{bmatrix}, \quad \tilde{B}_f = \begin{bmatrix} B_f + LD_f \\ B_f \end{bmatrix}$$

and $\tilde{B}_w = \begin{bmatrix} B_d + LD_d & \Delta B + L\Delta D \\ B_d & B + \Delta B \end{bmatrix}$.

By taking Laplace transforms, it is easy to show that

$$r(s) \ = \ \tilde{T}_{rf}(s)f(s) + \tilde{T}_{rw}(s)w(s), \tag{4}$$

where

$$\tilde{T}_{rf}(s) \ \stackrel{s}{=} \ \left[\begin{array}{c|c} \tilde{A} & \tilde{B}_f \\ \hline \tilde{H} & HD_f \end{array}\right] \in \mathcal{R}(s)^{n_f\times n_f}, \tag{5}$$

$$\tilde{T}_{rw}(s) \ \stackrel{s}{=} \ \left[\begin{array}{c|c} \tilde{A} & \tilde{B}_w \\ \hline \tilde{H} & \tilde{H}_w \end{array}\right] \in \mathcal{R}(s)^{n_f\times(n_d+n_u)}, \tag{6}$$

are the transfer matrices from faults and w to the residual, respectively. Note that the dynamics of the residual signal depend not only on f, d and u but also on the state x.

In robust FDI, we would ideally like to solve the following optimization problem:

$$\begin{array}{ll} \underset{\substack{\left\|\tilde{T}_{rf} - T_o\right\|_\infty \le \gamma_1 \\ \left\|\tilde{T}_{rw}\right\|_\infty \le \gamma_2 \\ \|T_o\|_- \ge 1 \\ \tilde{A} \ is \ stable \\ T_o(s) \in S}}{minimize} & \alpha\gamma_1 + (1-\alpha)\gamma_2 \end{array} \tag{7}$$

where $\alpha \in [0,1]$ is a parameter and S is the set of all transfer matrices with a special structure, e.g., diagonal, block diagonal or triangular. The first and third constraints ensure a minimum level of sensitivity of the residual to fault signals. The second ensures that the filter is robust to disturbances and model uncertainties. The fourth ensures that the filter is stable while the fifth ensures fault isolation and where the structure depends on the nature of the fault. Unfortunately, this optimization is intractable mainly due to the difficulty in characterizing the set S.

It is clear that minimizing γ_2 will improve fault detection and it is obvious that minimizing γ_1 will improve fault isolation. According to the design requirements, by choosing a suitable α, we can vary the emphasis between fault detection and fault isolation since optimizing one will generally affect the performance of the other.

Our focus in this work is on solving (7) when S is the set of all positive diagonal matrices. We will therefore consider the simplest nontrivial version of (7):

Problem 2.1: Assume that (C, A) is detectable and $(A + \Delta A)$ is stable. Let α be given. The optimal robust FDI filter design is to find L and H to minimize $\alpha\gamma_1 + (1-\alpha)\gamma_2$ such that:

$$\tilde{A} \text{ is asymptotically stable} \tag{8}$$

$$\left\|\tilde{T}_{rf} - T_o\right\|_\infty \leq \gamma_1, \tag{9}$$

$$\left\|\tilde{T}_{rw}\right\|_\infty \leq \gamma_2, \tag{10}$$

$$\|T_o\|_- \geq 1, \tag{11}$$

where T_o has the following structure :

$$T_o = N = \mathrm{diag}(n_1, \ldots, n_{n_f}) \in \mathcal{R}^{n_f \times n_f}, \quad n_i > 0,\ \forall i. \tag{12}$$

Remark 2.1: The assumptions (C, A) detectable and $(A + \Delta A)$ stable are necessary to have $\tilde{A}$ stable.

III. Matrix inequality formulation

We consider in this section a matrix inequality formulation approach to handle Problem 2.1. The main idea is to express the inequalities (9) to (11) in a matrix inequality formulation using the bounded real lemma, then finding an upper-bound on all the terms containing the unknown matrices $(\Delta A, \Delta B, \Delta C, \Delta D)$ using the fact that $\Delta\Delta' \leq I$.

Consider first the inequality (10). With the help of the version of the bounded real lemma given in Lemma 3.1, we can derive a matrix inequality formulation as follows.

Lemma 3.1: Let $\tilde{T}_{rw}$ define in (6). There exists L and H such that $\tilde{A}$ is stable and $\left\|\tilde{T}_{rw}\right\|_\infty < \gamma_2$ if and only if there exist L, H and $P = P' \in \mathcal{R}^{2n\times 2n}$ such that $P > 0$ and

$$\begin{bmatrix} (\tilde{A})'P + P(\tilde{A}) & P(\tilde{B}_w) & \tilde{H}' \\ (\tilde{B}_w)'P & -\gamma_2 I & \tilde{H}_w' \\ \tilde{H} & \tilde{H}_w & -\gamma_2 I \end{bmatrix} < 0. \tag{13}$$

The solution of the matrix inequality in (13) is not tractable in its current form since it is nonlinear. In order to get a tractable solution, we restrict P to have the form: $P = diag(P_1, P_2)$.

Using the expression of $\tilde{T}_{rw}$ in (6), we can separate the terms involving modeling uncertainties from the other terms since the inequality (13) can be written as

$$\hat{T}_{rw} + \Delta T_{rw} < 0, \tag{14}$$

where

$$\hat{T}_{rw} = \begin{bmatrix} (A+LC)'P_1 + (*) & (*) & (*) & (*) & (*) \\ 0 & P_2 A + (*) & (*) & (*) & (*) \\ (B_d + LD_d)'P_1 & B_d' P_2 & -\gamma_2 I & (*) & (*) \\ 0 & B' P_2 & 0 & -\gamma_2 I & (*) \\ HC & 0 & HD_d & 0 & -\gamma_2 I \end{bmatrix}$$

and

$$\Delta T_{rw} = \begin{bmatrix} 0 & (*) & (*) & (*) & (*) \\ (\Delta A + L\Delta C)'P_1 & P_2\Delta A + (*) & (*) & (*) & (*) \\ 0 & 0 & 0 & (*) & (*) \\ (\Delta B + L\Delta D)'P_1 & \Delta B' P_2 & 0 & 0 & (*) \\ 0 & H\Delta C & 0 & H\Delta D & 0 \end{bmatrix}.$$

Where $(*)$ denotes terms readily inferred from symmetry. By using the expressions of the modeling errors in (2), it can be verified that

$$\Delta T_{rw} = F_w \Delta E_w + E_w' \Delta' F_w' \tag{15}$$

where

$$F_w = \begin{bmatrix} P_1(F_1 + LF_2) \\ P_2 F_1 \\ 0 \\ 0 \\ HF_2 \end{bmatrix} \text{ and } E_w = \begin{bmatrix} 0 & E_1 & 0 & E_2 & 0 \end{bmatrix}.$$

The next result uses the fact that $\Delta\Delta' \leq I$ to bound ΔT_{rw}.

Lemma 3.2: Let F, E and Δ, be matrices with appropriate dimensions. If $\Delta\Delta' \leq I$ then

$$F\Delta E + E'\Delta' F' \leq FF' + E'E$$

Proof. Now

$$(F\Delta - E')(F\Delta - E')' = F\Delta\Delta' F' - F\Delta E - E'\Delta' F' + E'E.$$

It follows that

$$F\Delta E + E'\Delta' F' = -(F\Delta - E')(F\Delta - E')' + F\Delta\Delta' F' + E'E$$

However,

$$\Delta\Delta' \leq I \quad \Longleftrightarrow \quad \exists Z \text{ s.t. } \Delta\Delta' = I - ZZ'.$$

Thus

$$F\Delta E + E'\Delta' F' \leq FF' + E'E$$

■

It follows from (14), (15) and Lemma 3.2 that

$$\hat{T}_{rw} + \Delta T_{rw} \leq \hat{T}_{rw} + F_w F_w' + E_w' E_w.$$

Using a Schur complement type argument, it follows that

$$\bar{T}_{rw} < 0 \quad \Longrightarrow \quad \left\|\tilde{T}_{rw}\right\|_\infty < \gamma_2, \tag{16}$$

where

$$\bar{T}_{rw} = \begin{bmatrix} \hat{T}_{rw} + E_w' E_w & F_w \\ F_w' & -I \end{bmatrix}.$$

By using (12), it is easy to show that the inequality (9) is equivalent to

$$\left\| \left[\begin{array}{c|c} \tilde{A} & \tilde{B}_f \\ \hline \tilde{H} & HD_f - N \end{array} \right] \right\|_\infty < \gamma_1.$$

We can handle this inequality using the same procedure as that for inequality (10). Thus

$$\bar{T}_{rf} < 0 \quad \Longrightarrow \quad \left\|\tilde{T}_{rf} - T_o\right\|_\infty < \gamma_1, \tag{17}$$

where

$$\bar{T}_{rf} = \begin{bmatrix} \hat{T}_{rf} + E_f' E_f & F_f \\ F_f' & -I \end{bmatrix},$$

$$\hat{T}_{rf} = \begin{bmatrix} P_3(A+LC)+(*) & (*) & (*) & (*) \\ 0 & P_4 A+(*) & (*) & (*) \\ (B_f+LD_f)' P_3 & B_f' P_4 & -\gamma_1 I & (*) \\ HC & 0 & HD_f - N & -\gamma_1 I \end{bmatrix},$$

$$F_f = \begin{bmatrix} P_3(F_1 + LF_2) \\ P_4 F_1 \\ 0 \\ HF_2 \end{bmatrix} \text{ and } E_f = \begin{bmatrix} 0 & E_1 & 0 & 0 \end{bmatrix}.$$

It is easy to express the inequality (11) as a matrix inequality:

$$\begin{aligned} \|T_o\|_- \geq 1 \quad &\iff \quad \|N\|_- \geq 1 \\ &\iff \quad I_{n_f} - N \leq 0 \end{aligned} \tag{18}$$

The last inequality comes from the fact that N is diagonal and positive.

By using (16), (17) and (18), we can now derive a (nonlinear) matrix inequality formulation of Problem 2.1 as follows:

Problem 3.1: Let $\bar{T}_{rf}, \bar{T}_{rw}$ be as defined previously. Assume that (C, A) is detectable and let α be given. The optimal robust FDI filter design is to find L, H, N and a symmetric matrix $P = diag(P_1, P_2, P_3, P_4) \in \mathcal{R}^{4n}$ to minimize $\alpha\gamma_1 + (1-\alpha)\gamma_2$ such that:

$$P > 0, \tag{19}$$

$$\bar{T}_{rf} < 0, \tag{20}$$

$$\bar{T}_{rw} < 0, \tag{21}$$

and

$$I_{n_f} - N \leq 0, \tag{22}$$

where

$$N = \mathrm{diag}(n_1, \ldots, n_{n_f}) \in \mathcal{R}^{n_f \times n_f}, \ n_i > 0, \ \forall i. \tag{23}$$

IV. FDI FILTER DESIGN VIA LMI

In this section we give a state-space solution by formulating a linear matrix inequality (LMI). The inequality (22) is linear while both inequalities (20) and (21) are quadratic and therefore cannot be easily solved. The next theorem gives sufficient conditions for the existence of L and H in the form of LMIs.

Theorem 4.1: Assume that (C, A) is detectable. Let γ_1 and γ_2 be given. There exist $L \in \mathcal{R}^{n \times n_y}$ and $H \in \mathcal{R}^{n_f \times n_y}$ such that the specifications (8), (9), (10) and (11) in Problem 2.1 are satisfied if there exist $Q \in \mathcal{R}^{n \times n_y}$, $H \in \mathcal{R}^{n_f \times n_y}$, $N \in \mathcal{R}^{n_f \times n_f}$ and a symmetric matrix $P = diag(P_1, P_2, P_4) \in \mathcal{R}^{3n}$ such that

$$P > 0, \tag{24}$$

$$\begin{bmatrix} P_1 A+QC+(*) & (*) & (*) & (*) & (*) \\ 0 & P_4 A+(*)+E_1' E_1 & (*) & (*) & (*) \\ B_f' P_1 + D_f' Q' & B_f' P_4 & -\gamma_1 I & (*) & (*) \\ HC & 0 & HD_f - N & -\gamma_1 I & (*) \\ F_1' P_1 + F_2' Q' & F_1' P_4 & 0 & F_2' H' & -I \end{bmatrix} < 0 \tag{25}$$

$$\begin{bmatrix} P_1 A+QC+(*) & (*) & (*) & (*) & (*) & (*) \\ 0 & P_2 A+(*)+E_1' E_1 & (*) & (*) & (*) & (*) \\ B_d' P_1 + D_d' Q' & B_d' P_2' & -\gamma_2 I & (*) & (*) & (*) \\ 0 & B' P_2 + E_2' E_1 & 0 & -\gamma_2 I + E_2' E_2 & (*) & (*) \\ HC & 0 & HD_d & 0 & -\gamma_2 I & (*) \\ F_1' P_1 + F_2' Q' & F_1' P_2 & 0 & 0 & F_2' H' & -I \end{bmatrix} < 0 \tag{26}$$

and

$$I_{n_f} - N \leq 0, \tag{27}$$

where

$$N = \mathrm{diag}(n_1, \ldots, n_{n_f}) \in \mathcal{R}^{n_f \times n_f}, \quad n_i > 0, \ \forall i.$$

Furthermore, if (25) and (26) are feasible, we can construct L as :

$$L = P_1^{-1} Q.$$

Proof.
By setting $P_1 = P_3$ and $Q = P_1 L$ in (20) and (21), we get (25) and (26). Then (8), (9) and (10) are satisfied if the LMIs have a feasible solution. The corresponding filter design is given by extracting $L = P_1^{-1} Q$ and H from the LMIs. ■

V. NUMERICAL EXAMPLE

To illustrate the effectiveness of the proposed fault detection and isolation filter scheme, an example is considered in this section. The GE-21 jet engine state-space model [8] is given as

$$\begin{aligned} A &= \begin{bmatrix} -3.370 & 1.636 \\ -0.325 & -1.896 \end{bmatrix}, \\ B &= \begin{bmatrix} 0.586 & -1.419 & 1.252 \\ 0.410 & 1.118 & 0.139 \end{bmatrix}, \\ C &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0.731 & 0.786 \end{bmatrix}, \\ D &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.267 & -0.025 & -0.146 \end{bmatrix}. \end{aligned}$$

We suppose that this system is subject to two disturbances and three potential faults. Here, the setup randomly generated is given by

$$B_d = \begin{bmatrix} 0.1379 & 0.6204 \\ -0.3145 & 0.1602 \end{bmatrix},$$
$$D_d = \begin{bmatrix} 0.2618 & 0.1745 \\ 0.1047 & -0.1461 \\ 0.3277 & 0.1737 \end{bmatrix},$$
$$B_f = \begin{bmatrix} -1.1679 & 0.3977 & -1.0846 \\ -0.2444 & 0.5619 & -0.1957 \end{bmatrix},$$
$$D_f = \begin{bmatrix} -0.3986 & -0.8364 & -0.4238 \\ -1.0324 & -0.0069 & 1.9389 \\ -2.8553 & 0.6071 & 0.9040 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}', \quad F_2 = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}',$$
$$E_1 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}.$$

We implemented the algorithm in Theorem 4.1 in MATLAB to minimize $\alpha\gamma_1 + (1-\alpha)\gamma_2$ and compute L and H. We have chosen $\alpha = 0.5$ as a compromise between fault detection and fault isolation. Using the algorithm of Section IV we get

$$L = \begin{bmatrix} -0.3084 & 0.8750 & -0.7369 \\ 0.5500 & 0.4118 & -0.3195 \end{bmatrix},$$
$$H = \begin{bmatrix} -0.2533 & 0.1212 & -0.3615 \\ -1.0278 & -0.3313 & 0.2542 \\ -0.1230 & 0.5671 & -0.1821 \end{bmatrix},$$
$$N = I_3, \quad \gamma_1 = 0.0552, \quad \gamma_2 = 0.3677. \tag{28}$$

By comparing the values of γ_1 and γ_2 given in (28) with Figure 1, we can say that the filter will achieve fault detection and isolation.

In order to show that our filter is robust against disturbances and model uncertainties, we introduce the uncertainties defined by the matrices (F_1, F_2, E_1, E_2), as well as two disturbances. Simulated through MATLAB and SIMULINK, these disturbances are band limited white noise with power 0.01 (zeroth-order hold with sampling time $0.1\,s$), and positive jump from the 6^{th} second. Fault f_1, simulated by a unit positive jump is connected from the 14^{th} second. Fault f_2, simulated by a unit negative jump and fault f_3, simulated by a soft bias ($slope = 0.5$), are both connected from the 22^{th} second. The input u is taken as a periodic signal.

Figure 2 gives the residual responses, where each fault can be readily distinguished from the others and the disturbances. It is worth noting that , in order to verify the effectiveness of the optimal filter, similar amplitudes for the disturbance and faults are assumed, which is quite demanding from the practical point of view.

This example makes clear that the designed filter satisfies the performance requirement of rapid detection and direct fault isolation which is sufficiently robust against disturbances and modeling errors.

Notice that the time responses clearly show that it is possible to set thresholds that allow us to distinguish between faults and disturbances.

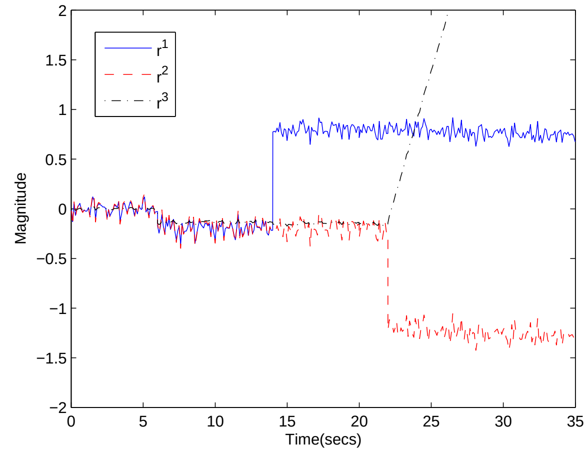

Figure 2: Time response of the residual

VI. Summary

In this paper, we have considered a robust fault detection and isolation problem using a matrix inequality formulation approach. We derive a sufficient condition for solvability of the robust FDI problem in the form of an LMI formulation, and furthermore, give a construction of a stable FDI filter that bounds the influence of the disturbances and model uncertainties on the residual signal measured in terms of the $\mathcal{H}_\infty$–norm. Our scheme implies that each element of the residual is only sensitive to a specified potential fault and therefore can handle multiple faults (where faults might occur at the same time). A jet-engine example is given to illustrate our algorithm.

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