Multiple-Antenna Differential Lattice Decoding

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***Abstract*—From a lattice viewpoint, Clarkson, Sweldens and Zheng significantly reduced the complexity of multiantenna differential decoding. Their approximate decoding algorithm, however, has not unleashed the full potential of lattice decoding. In this paper, we present several improved algorithms, generally referred to as differential lattice decoding (DLD), for multiantenna communication. We first analyze two distinct approximate DLD algorithms, and then develop an algorithm that exactly finds the closest lattice point in the Euclidean space. This exact DLD is subsequently augmented by local search to compensate for the remaining approximation. The small amount of extra complexity of the exact or augmented DLD is rewarded by a clear performance gain. We find that employing basis reduction is very effective to reduce the overall decoding complexity for high lattice dimensions. Moreover, the dimension of the lattice defined in this paper is independent of the number of receive antennas, which results in not only lower complexity, but also better performance for a multiantenna receiver.**

***Index Terms*—Basis reduction, differential modulation, lattice decoding, multiantenna communication, sphere decoding.**

I. Introduction

ONE OF THE MAIN motivations of using multiple-input–multiple-output (MIMO) wireless communication is to support very high data rates. This however makes a naive maximum-likelihood decoder (MLD) quickly become impractical, as its complexity grows exponentially with the data rate. Consequently, it is crucial to develop efficient decoding algorithms for high-rate MIMO systems. Lattice decoding has recently received great interests in MIMO research due to its low average complexity for many communication problems. See [1]–[3] for an overview of lattice decoding and [4] and [5] for early applications to communications. Generally, lattice decoding consists of two major stages—basis reduction and enumeration of nearby lattice points, the latter of which is often called sphere decoding in communication literature following [5]. The first stage selects a short and fairly orthogonal basis, usually by using the Lenstra–Lenstra–Lovasz (LLL) algorithm [6], while the second finds the closest lattice point from those falling inside a sphere centered at the query point [7]–[9].

Like most techniques for coherent MIMO communication, the principle of lattice decoding is also applicable to noncoherent systems where channel estimation is difficult or uneconomical. Differential space–time modulation (DSTM) [10], [11] is a noncoherent MIMO scheme specially tailored for continuously fading channels. Notably, Clarkson *et al.* [12] gave an elegant lattice formulation of diagonal DSTM and developed fast, approximate differential decoding (DD)[1] based on the LLL basis reduction. The achieved complexity is polynomial in the number of transmit antennas. This technique has been extended to some nondiagonal constellations [13], [14] and to decision-feedback differential detection (DF-DD) [15]. The concern of these papers is the application rather than the investigation of the fast decoding algorithm itself.

In this paper, we present several improved fast algorithms of conventional DD and DF-DD for diagonal DSTM by employing the principle of lattice decoding, which are generally referred to as differential lattice decoding (DLD). As pointed out in [12], there are three approximations associated with component wise rounding in an LLL-reduced lattice. We eliminate two of these by using exact DLD which finds the closest lattice point exactly. The remaining approximation of the cosine function is, however, inherent with the lattice formulation. We therefore augment DLD by local search with the cosine measure, which gives performance indistinguishable from MLD. It is shown that both exact and augmented DLD can be sped up significantly if the basis is LLL-reduced. Moreover, the decoders under our formulation always work with an n_T-dimensional (n_T-D) lattice for a system with n_T transmitter antennas and n_R receiver antennas, while the decoder in [12] needs to define an $n_T n_R$-D lattice. Not only does this formulation reduce the decoding complexity, the performance also improves since the approximations are less accurate for higher dimensional lattices.

The paper is organized as follows. Section II gives the system model of DSTM and a brief review of the lattice formulation for conventional DD and DF-DD. The improved formulation of an n_T-D lattice for multiple receive antennas is integrated with the review. Approximate, exact, and augmented DLD are presented in Section III. Here, we clearly distinguish between two approximate DLD approaches, which is not addressed in [12]. Numerical results on the performance and complexity of

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[1]In line with the literature of lattice decoding, "decoding" and "detection" are used interchangeably in this paper.

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various decoders are reported in Section IV. Finally, conclusions and discussion are given in Section V.

Notations: Throughout this paper, matrices (vectors) are represented in bold upper (lower) case, $\mathbf{I}_n$ denotes the n-by-n identity matrix, $(\cdot)^*$ complex conjugate, $(\cdot)^T$ transpose, $(\cdot)^H$ Hermitian transpose, $\mathrm{tr}(\cdot)$ trace, $\mathrm{diag}(\cdot)$ denotes forming a diagonal matrix from a vector, $\|\mathbf{A}\|^2 = \mathrm{tr}(\mathbf{A}\mathbf{A}^H)$ denotes the Frobenius norm, and $\lceil \cdot \rfloor$ denotes rounding toward the nearest integer.

II. Lattice Formulation

A. System Model

Consider an $n_T \times n_R$ DSTM system over a flat, time-selective fading channel, where the time selectivity is described by the Jakes model [16] for instance. Transmitted signals are organized into an $n_T \times n_T$ matrix $\mathbf{S}[\tau]$, where row indexes represent different antennas and column indexes represent time instants $\tau n_T, \ldots, \tau n_T + n_T - 1$. The matrix is properly normalized so that the average power of each column is one. The total transmitted power, therefore, does not depend on the number of transmit antennas.

The constellation $\mathcal{G}$ of a rate-R_c (bits/channel use) DSTM system is comprised of $L = 2^{R_c n_T}$ unitary matrices $\mathbf{G}_l$, $l \in \{0, \cdots, L-1\}$, of size $n_T \times n_T$ each. The $R_c n_T$ bits to be transmitted at time instant τn_T are mapped to an L-ary symbol $a[\tau]$, which selects an element $\mathbf{G}_{a[\tau]}$ from $\mathcal{G}$. Signal matrices are then differentially encoded as $\mathbf{S}[\tau] = \mathbf{S}[\tau-1]\mathbf{G}_{a[\tau]}$ with $\mathbf{S}[0] = \mathbf{I}_{n_T}$. The diagonal constellations of Hochwald and Sweldens [11] have the form

$$\mathbf{G}_l = (\mathbf{G}_1)^l, \text{ where } \mathbf{G}_1 = \mathrm{diag}\left[e^{\frac{j2\pi u_1}{L}}, \cdots, e^{\frac{j2\pi u_{n_T}}{L}}\right], \quad 0 \le l < L$$

and the integers $u_m \in \{0, 1, \ldots, L-1\}$ for $1 \le m \le n_T$. $\mathcal{G}$ is a cyclic group under matrix multiplication. One may set $u_1 = 1$ without loss of generality. The generator $\mathbf{u} = [1, u_2, \ldots, u_{n_T}]$ is searched to maximize the diversity product [11].

Since there is a single transmit antenna active at each time instant for diagonal constellations, we define an $n_R \times n_T$ channel matrix $\mathbf{H}[\tau]$ where the (i,j)th entry $h_{i,j}[\tau]$ denotes the fading coefficient between receive antenna i and transmit antenna j at time instant $\tau n_T + j$. Accordingly, the received signals can be expressed by

$$\mathbf{Y}[\tau] = \sqrt{\rho}\mathbf{H}[\tau]\mathbf{S}[\tau] + \mathbf{W}[\tau] \tag{1}$$

where $\mathbf{W}[\tau]$ is the corresponding noise matrix. The entries of both $\mathbf{H}[\tau]$ and $\mathbf{W}[\tau]$ are independent across space and time, and are identically complex normal $\mathcal{CN}(0,1)$ distributed. Because of the power normalization, ρ is the average signal-to-noise ratio (SNR) at each receive antenna. In the Jakes model, the correlation after k time samples of $h_{i,j}[\tau]$ is given by $J_0(2\pi f_d n_T k)$, where $J_0(.)$ is the zeroth-order Bessel function of the first kind, and f_d is the normalized Doppler shift with respect to the scalar symbol period. Note that with this signal model for diagonal constellations, the effective Doppler shift of the fading process is multiplied by a factor of n_T.

B. Improved Problem Formulation

The maximum-likelihood (ML) DD for diagonal constellations is given by

$$\begin{aligned}\hat{a}^{\mathrm{ML}}[\tau] &= \arg\min_l \|\mathbf{Y}[\tau] - \mathbf{Y}[\tau-1]\mathbf{G}_l\|^2 \\ &= \arg\max_l \mathrm{Re}\left[\mathrm{tr}\left(\mathbf{G}_l\mathbf{Y}^H[\tau]\mathbf{Y}[\tau-1]\right)\right] \\ &= \arg\max_l \sum_{m=1}^{n_T}\sum_{k=1}^{n_R} \mathrm{Re} \\ &\quad \times \left[e^{\frac{j2\pi u_m l}{L}} y^*_{k,m}[\tau] y_{k,m}[\tau-1]\right].\end{aligned} \tag{2}$$

Noticing that $e^{j2\pi u_m l/L}$ does not depend on index k, we may rewrite the MLD as

$$\hat{a}^{\mathrm{ML}}[\tau] = \arg\max_l \sum_{m=1}^{n_T} \mathrm{Re} \times \left[\left(\sum_{k=1}^{n_R} y_{k,m}[\tau] y^*_{k,m}[\tau-1]\right) e^{-\frac{j2\pi u_m l}{L}}\right].$$

Define

$$\begin{aligned}A_m &= \left|\sum_{k=1}^{n_R} y_{k,m}[\tau] y^*_{k,m}[\tau-1]\right|^{\frac{1}{2}} \\ \phi_m &= \frac{L}{2\pi}\arg\left(\sum_{k=1}^{n_R} y_{k,m}[\tau] y^*_{k,m}[\tau-1]\right)\end{aligned} \tag{3}$$

where $\arg(\cdot)$ has range $[-\pi, \pi)$ so that $\phi_m \in [-L/2, L/2)$. Then, we have

$$\hat{a}^{\mathrm{ML}}[\tau] = \arg\max_l \sum_{m=1}^{n_T} A_m^2 \cos\left[\frac{(u_m l - \phi_m)2\pi}{L}\right]. \tag{4}$$

Apparently, the complexity of MLD is proportional to L. In other words, it is exponential in the number of transmit antennas and the rate. Equation (4) is formally the same as [12, eq. (11)], but integrates the case of $n_R > 1$. This reduces the number of summands by a factor of n_R (cf. [12, eq. (24)]).

To derive a fast decoder, a lattice viewpoint is adopted in [12]. Since the cosine function is 2π periodic, the argument can be restricted to $[-\pi, \pi)$ without loss of generality. The argument of the cosine function in (4) can, thus, be rewritten as $[(u_m l - \phi_m)\mathrm{mod}^* L]2\pi/L$, where mod^* takes values in $[-L/2, L/2)$. The vectors $l\mathbf{u}\,\mathrm{mod}^* L$ for $l = 0, \ldots, L-1$ form a finite integer lattice. Furthermore, an approximation $\cos\alpha \approx 1 - \alpha^2/2$ is made in [12]. This brings the lattice decoder into a Euclidean space. The best solution in the Euclidean space

$$\hat{a}^{\mathrm{eucl}}[\tau] = \arg\min_l \sum_{m=1}^{n_T} \left[(A_m u_m l - A_m\phi_m)\mathrm{mod}^* A_m L\right]^2 \tag{5}$$

is supposed to be a good approximation of the ML decision, because the cosine approximation is locally accurate near the maximum at $\alpha = 0$. If we define a basis

$$\mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, \cdots, \mathbf{b}_{n_T}] = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ A_2 u_2 & A_2 L & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{n_T} u_{n_T} & 0 & \cdots & A_{n_T} L \end{bmatrix}$$

the point set $\{\mathbf{Bx} : \mathbf{x} \in \mathbb{Z}^{n_T}\}$ will form an *infinite lattice* in $\mathbb{R}^{n_T}$. Denote by $\mathbf{t}$ the target vector with components $A_m\phi_m$. The decoding problem (5) can be recast into a standard closest (lattice) vector problem (CVP), in which an integer vector $\hat{\mathbf{x}} \in \mathbb{Z}^{n_T}$ is to be found such that $\|\mathbf{Bx}-\mathbf{t}\|^2$ is minimized. It is seen that $\mathrm{mod}^* \ A_1 L$ is artfully omitted in $\|\mathbf{Bx}-\mathbf{t}\|^2$. Such decoding in an infinite lattice has the convenient feature that no boundary control is needed. Accordingly, the decision is given by

$$\hat{a}^{\mathrm{latt}}[\tau] = \hat{x}_1 \bmod L \tag{6}$$

and the omission of $\mathrm{mod}^* \ A_1 L$ does not affect the exactness of the solution to (5).

In essence, the ML Voronoi region is approximated by a polytope in the above lattice formulation. The approximation is quite accurate in low dimensions, but may be less accurate in high dimensions. It is worth mentioning that in our formulation, the lattice *always* has dimension n_T, regardless of the value of n_R. This is in contrast with [12, (24)], where an $n_T n_R$-D lattice would be needed. Thanks to the notation (3), we know an n_T-D lattice is sufficient to define the decoding problem [cf. (4)]. Therefore, our formulation will improve the performance, as well as the speed of DLD for a multiantenna receiver.

C. DF-DD

Because of the n_T-fold Doppler shift, conventional DD easily suffers from an irreducible error floor in fast-fading channels. Linear predictive DF-DD is an appealing technique of improvement for diagonal constellations. To mitigate the error floor, DF-DD makes use of the observations during the past few time epochs [15]. More precisely, it takes the form

$$\hat{a}[\tau] = \arg\max_{l} \mathrm{Re}\left[\mathrm{tr}\left(\mathbf{G}_l \mathbf{Y}^H[\tau] \sum_{n=1}^{N} p_n \mathbf{Y}[\tau-n] \times \prod_{i=1}^{n-1} \mathbf{G}_{\hat{a}[\tau-i]}\right)\right] \tag{7}$$

where N is the prediction order, p_n for $n = 1,\ldots,N$ are predictor taps, and $\prod_{i=1}^{n-1} \mathbf{G}_{\hat{a}[\tau-i]} \triangleq \mathbf{I}_{n_T}$ if $n = 1$. Clearly, the lattice formulation is valid for DF-DD verbatim, if we replace $\mathbf{Y}[\tau-1]$ in (2) by the sum in (7) that corresponds to linear prediction. Again, an n_T-D lattice suffices and there is no need in going to an $n_T n_R$-D lattice as in [15].

III. Differential Lattice Decoding

In this section, we present several procedures to approximately or exactly find the closest lattice point $\hat{\mathbf{x}} \in \mathbb{Z}^{n_T}$ and give a further improvement when the performance loss due to the cosine approximation is not negligible. All procedures are applicable to both conventional DD and DF-DD obviously. Since the basis $\mathbf{B}$ is a function of A_m's and in turn by received signal matrices [cf. (3)], it constantly changes over time, and basis reduction and QR decomposition have to be repeated for each τ. This is an important distinction from the ordinary application of lattice decoding in quasi-static fading.

A. Approximate DLD

Basis reduction alone can be used to solve CVP approximately. Ideally, if the basis were orthogonal, simple rounding off would find the closest lattice point. This is the rare case in reality, and basis reduction attempts to select a nearly orthogonal and short basis. The celebrated LLL algorithm finds a basis within an exponential factor from the shortest in polynomial time [6], which requires $O(n_T^4)$ arithmetic operations. The reduced and unreduced bases are related by $\mathbf{B}_r = \mathbf{BU}$, where $\mathbf{U} \in \mathbb{Z}^{n_T \times n_T}$ is a unimodular matrix with determinant $\det(\mathbf{U}) = \pm 1$.

Babai [17] gave two procedures for solving the approximate CVP in an LLL-reduced lattice. In the context of DLD, Babai's procedures can be rephrased as follows. Let the Gram-Schmidt orthogonalization of the reduced basis $\mathbf{B}_r$ be $\mathbf{B}_r = \hat{\mathbf{B}}_r[\mu_{ij}]^T$ and $\boldsymbol{\zeta} = \hat{\mathbf{B}}_r^{-1}\mathbf{t}$. Then, the closest vector, up to a certain factor is, respectively, given by the following.

- Rounding off (LLL-ZF): $\hat{\mathbf{x}} = \mathbf{U}\lceil \mathbf{B}_r^{-1}\mathbf{t} \rfloor$.
- Nearest plane (LLL-SIC): $\hat{\mathbf{x}} = \mathbf{U}\hat{\mathbf{z}}$, where $\hat{z}_{n_T} = \lceil \zeta_{n_T} \rfloor$ and $\hat{z}_j = \lceil \zeta_j - \sum_{i=j+1}^{n_T} \hat{z}_i \mu_{ij} \rfloor$ for $j = n_T - 1,\ldots,1$.

The complexity of both procedures is negligible in comparison with LLL reduction. It can be seen that the rounding off procedure is equivalent to zero-forcing (ZF) detection in communications, while the nearest plane procedure is equivalent to nulling and successive interference cancellation (SIC) detection of the vertical Bell Labs layered space–time architecture (V-BLAST) without ordering. It is shown in [2] that V-BLAST ordering cannot improve the performance over the natural ordering of the nearest plane procedure for an LLL-reduced lattice, although it is beneficial for an unreduced lattice. This is because ordering is equivalent to a series of swapping operations, which have already been done in LLL reduction. As such, we will not consider ordering for approximate DLD in a reduced lattice.

Due to the incorporation of decision feedback, LLL-SIC has better performance than LLL-ZF. Thus, it is important to make a clear distinction between the two decoders. The "component wise rounding" in [12] is just Babai's rounding off procedure, though this is not explicitly pointed out therein.[2] In the rounding off procedure, the polyhedral Euclidean Voronoi cell is further approximated by a parallelepiped [12, Fig. 3]. In the nearest plane procedure, it is better fitted by a cuboid, as will be demonstrated in the following example.

1) Example: We reexamine the case considered in [12], where $n_T = 2$, $\mathbf{u} = [1\ 9]^T$, $L = 32$, and $A_1 = A_2 = 1$. Fig. 1 shows the decision regions of the origin for different decoders. The Euclidean Voronoi cell is the solid-line hexagon. Suppose the shortest basis vectors $[4\ 4]^T$ and $[3\ \ -5]^T$ are identified successfully. Then, the dash-dotted diamond is the decision region for the ZF decoder. The two orthogonal vectors resulting from Gram–Schmidt orthogonalization are given by $[4\ 4]^T$ and $[4\ \ -4]^T$. Thus, the SIC decision region is the dotted rectangle (in fact a square in this special case). Apparently, the SIC decision region has more overlap with the Euclidean Voronoi

[2]While [12, eq. (22)] is a property of the nearest plane procedure, the hat over $\mathbf{b}_i$ there might be a typo.

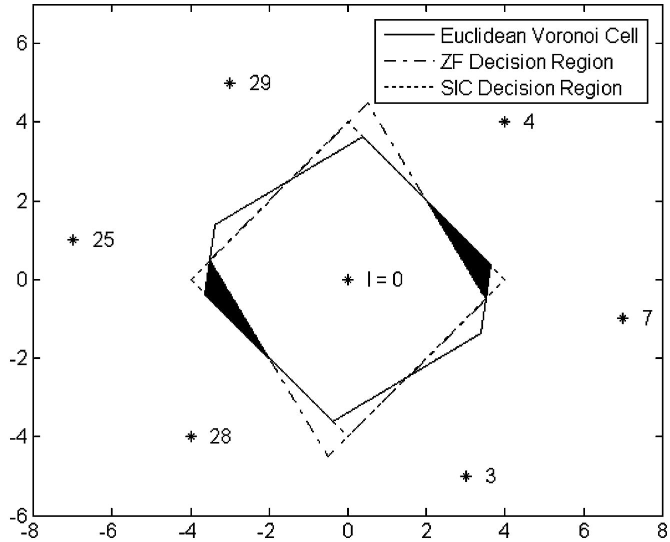

Fig. 1. Decision regions of the origin ($l = 0$) for MLD, ZF, and SIC. The two small filled triangles represent the extra overlap due to SIC.

cell, thereby resulting in improved performance. The two small filled triangles represent the additional overlap due to SIC.

The decision regions such as those in Fig. 1 bear useful information. Study of the minimum squared Euclidean distance (MSED) can tell how much LLL-ZF/SIC is in proximity to MLD. In the Appendix, we show that, for any LLL-reduced lattice, the MSED loss (in decibels) is linear with the dimension of the lattice for both ZF and SIC, while SIC has a slower increasing rate. Loosely speaking, this means that the error-rate curve of approximate DLD will parallel that of MLD at high SNR, with a horizontal translation (in decibels) linear with the dimension. In view of this analysis, the superiority of our n_T-D model in Section II-B is clear, since a lower dimension means less performance loss.

B. Exact DLD

As noted in [12], besides the cosine approximation, there are two other approximations associated with LLL-ZF or LLL-SIC that make it a suboptimum lattice decoder: The Euclidean Voronoi cell is approximated by a parallelepiped or cuboid, and the LLL algorithm does not necessarily find the shortest basis. Here we apply sphere decoding to solve the CVP exactly, which has low average complexity [4], [18] for many communication problems at high SNR. In Fig. 1, the Euclidean Voronoi cell corresponds to the decision region of sphere decoding. In the original form of sphere decoding, the Pohst–Fincke strategy [7], [8] enumerates all lattice points satisfying $\|\mathbf{Bx} - \mathbf{t}\|^2 \leq R^2$, where R denotes the chosen radius of the sphere. To determine the closest point, it is typical to decrease R accordingly once a closer point is obtained. Since the Schnorr–Euchner strategy [9] is more efficient in *closest lattice point search*, it is adopted here for DLD.

In sphere decoding, a basis is first transformed into its upper triangular representation by QR decomposition. Alternatively, it can be transformed into the lower triangular representation by QL decomposition. As the DSTM basis $\mathbf{B}$ is already lower triangular, it looks plausible to decode on $\mathbf{B}$ directly. This, however, turns out to be extremely inefficient. This is because A_1, the first diagonal element of $\mathbf{B}$, is typically much smaller than the other diagonal elements $A_m L$. It easily leads to early errors that are quite wasteful. Consequently, this method will not be considered, and we follow the usual QR decomposition. Noting that $\mathbf{B}$ is a square matrix, we have the QR decomposition $\mathbf{B} = \mathbf{QR}$, where $\mathbf{Q}$ is an orthogonal matrix with positive diagonal elements, and $\mathbf{R}$ is upper triangular. The constraint $\|\mathbf{Bx} - \mathbf{t}\|^2 \leq R^2$ can be put in an equivalent form

$$\|\mathbf{Rx} - \mathbf{t}'\|^2 \leq R^2, \quad \text{where} \quad \mathbf{t}' = \mathbf{Q}^T\mathbf{t}.$$

This can be solved successively by back substitution, starting at the last element of $\mathbf{x}$. Moreover, since the lattice is infinite, no boundary control is necessary and the subroutine of the Schnorr–Euchner search given in [1] can be applied readily. The initial radius of the Schnorr–Euchner strategy can be set to infinity, and the first point found corresponds to the SIC decision (also referred to as the Babai point). In the Schnorr–Euchner strategy, every time when a closer lattice point is found, it is stored as a potential output point, the radius R is decreased to the distance between this point and the query point, and the algorithm backtracks without restarting. The final output of the decoder gives the closest lattice point.

There are a number of ways of preprocessing or ordering to speed up the sphere decoder [1]–[3]. We compare their applications to DLD in the following.

1) Basis Reduction: Basis reduction is an essential preprocessing stage of lattice decoding, which improves the speed of sphere decoding drastically. For a finite lattice, basis reduction is sometimes considered inconvenient, as the boundary control is complicated in the reduced lattice [3]. Such an issue does not exist in an infinite DSTM lattice. It raises another issue though, that is, basis reduction needs to be reperformed for every incoming symbol. Consequently, it has a significant impact on the overall complexity of the DLD, unlike the usual scenario where the channel remains constant for a long time so that the complexity of basis reduction is negligible. In high dimensions, basis reduction will usually constitute a major computational burden of DLD. Nonetheless, it is still the fastest solution in our context for large values of n_T, as will be shown in the next section. The reason is that the subsequent sphere decoding will typically be much faster, thanks to the nice structure of a reduced basis. It only adds a fraction of complexity to LLL-ZF or LLL-SIC. Suppose $\hat{\mathbf{x}}_r$ is the closest vector on the reduced basis $\mathbf{B}_r$, then its representation in the original lattice is given by $\hat{\mathbf{x}} = \mathbf{U}\hat{\mathbf{x}}_r$.

2) Ordering: For an *unreduced* lattice, the order in which the components of $\mathbf{x}$ are decoded may strongly influence the decoding speed. Ordering corresponds to choosing a column permutation matrix $\mathbf{\Pi}$ such that the new basis $\mathbf{B}' = \mathbf{B\Pi}$ has some desirable properties. Suppose $\hat{\mathbf{x}}'$ is the closest vector on the ordered basis $\mathbf{B}'$, then we have the relation $\hat{\mathbf{x}} = \mathbf{\Pi}\hat{\mathbf{x}}'$. Again, since the basis $\mathbf{B}$ changes every time, ordering needs to be redone for each incoming received signal matrix. Generally, the computational overhead of ordering is not negligible. The V-BLAST ordering is widely applied prior to sphere decoding for MIMO systems. To reduce the $O(n_T^4)$ complexity of the original V-BLAST ordering, several $O(n_T^3)$ methods have been proposed in literature (see, e.g., [19] and references therein). As will be demonstrated in the next section, while ordering itself is faster than LLL reduction, the latter results in substantially faster overall decoding in high dimensions.

Fincke and Pohst [8] suggest column ordering according to norms between basis reduction and sphere decoding. We observe little acceleration of sphere decoding, however, if the basis has been *LLL-reduced*. This observation agrees with coherent lattice decoding [2], and can also be interpreted as the consequence of the fact that ordering is a swapping-only operation. This ordering method is applied in [3] to an unreduced lattice, which is less effective than V-BLAST ordering.

C. Augmented DLD

Although the sphere decoder finds the closest lattice point in the Euclidean space, the cosine approximation remains to be a suboptimum factor, especially in high dimensions. Since the cosine approximation is locally accurate, a local search with the cosine measure would largely compensate for the suboptimality. This is tantamount to enumeration of *all lattice points* inside a sphere, which is exactly achieved by the Pohst–Fincke algorithm [7], [8]. When doing this, it will not shrink the radius R even if a closer point is found. The only distinction of our algorithm is that we make the enumeration for the one maximizing the MLD metric

$$\mathcal{M}_l \triangleq \sum_{m=1}^{n_T} A_m^2 \cos\left[\frac{(u_m l - \phi_m)2\pi}{L}\right].$$

It is easy to integrate the MLD metric into the Pohst–Fincke enumeration. Namely, each time a lattice point $\mathbf{x}_r$ is found, it is converted into the original lattice $\mathbf{x} = \mathbf{U}\mathbf{x}_r$ (if the lattice is reduced), $l = x_1 \bmod L$ is obtained, and $\mathcal{M}_l$ is compared to the maximum metric $\hat{\mathcal{M}}$ so far. If $\mathcal{M}_l$ is larger, $\hat{\mathcal{M}}$ is replaced by $\mathcal{M}_l$, and the most likely symbol $\hat{l}$ so far is replaced by l as well. After the enumeration terminates, $\hat{l}$ is the desired output. Note that the unimodular matrix $\mathbf{U}$ does not change with the particular point once the basis has been reduced.

The Schnorr–Euchner strategy can also be modified to enumerate all lattice points inside a sphere for one with the maximum MLD metric. When it finds a point (namely, the first element x_1 is reached) inside the initial radius R, it takes three different actions: 1) R is never changed; 2) it searches for all possible values of x_1 of in-sphere points rather than just the best one before backtracking; and 3) the pair $(\hat{l}, \hat{\mathcal{M}})$ is updated as described above. The Schnorr–Euchner strategy and the Pohst–Fincke strategy have similar complexity in enumeration of *all lattice points* inside a sphere. Either one can be applied to augmented DLD.

Augmented DLD differs from the list sphere decoder of [20] in that we do not maintain a list of candidates. We only store the most likely symbol $\hat{l}$ so far. In this way, we make full use of the lattice points inside the sphere. Then, it is crucial to select the proper initial radius R for augmented DLD. A too large value of R will slow down the decoder because many points need to be enumerated. On the other hand, if R is too small, then probably no other points will be found and the performance cannot be improved. To determine an appropriate value of R, let us examine

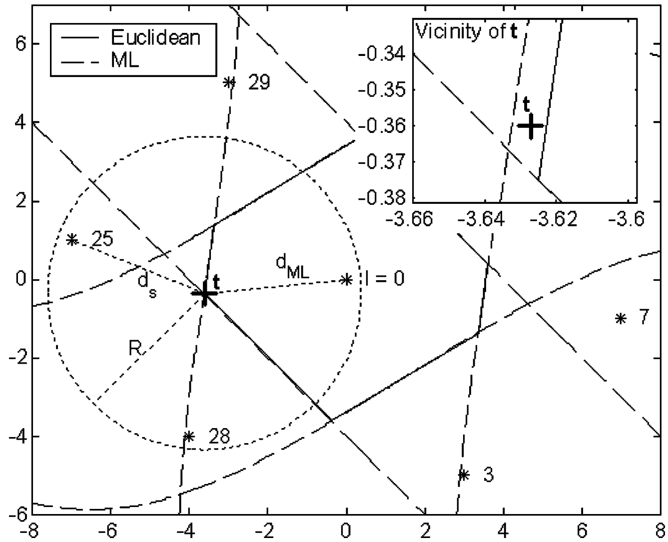

Fig. 2. Initial radius R of augmented DLD is not much larger than d_s. In the upper-right corner the vicinity of $\mathbf{t}$ is enlarged.

the scenario in which the exact DLD errs, whereas the MLD makes a correct decision. It is over here that the improvement of augmented DLD should take place if there is any. This scenario happens if the query point $\mathbf{t} \in \mathcal{V}^{\mathrm{ML}} \cap \overline{\mathcal{V}}^{\mathrm{eucl}}$, where $\mathcal{V}^{\mathrm{ML}}$ and $\mathcal{V}^{\mathrm{eucl}}$ denote the ML and Euclidean Voronoi cell, respectively.

1) Example: Fig. 2 demonstrates such a scenario in the lattice of Fig. 1. The ML Voronoi cell is the closeup of dashed lines, where two lattice points have equal likelihood (i.e., they are pairwise decision boundaries.) $\mathcal{V}^{\mathrm{ML}}$ and $\mathcal{V}^{\mathrm{eucl}}$ are virtually indistinguishable for this lattice (but they may differ more for certain channel realizations or in high dimensions.) For visual ease, the vicinity of the query point $\mathbf{t}$ is enlarged in the upper-right corner, which shows that the exact DLD decides in favor of $l = 25$, whereas the most likely point is in fact $l = 0$. Clearly, the distance d_{ML} between $\mathbf{t}$ and $l = 0$ is approximately equal to the distance d_s between $\mathbf{t}$ and $l = 25$. Accordingly, to include the ML decision in the sphere, the radius R does not need to be much larger than d_s.

Generally, the scenario only happens if $\mathbf{t}$ is near the boundary of $\mathcal{V}^{\mathrm{eucl}}$, since the cosine approximation is locally accurate. In other words, the most likely point is *not much further* from $\mathbf{t}$ than the sphere decoder output. Thus, R only has to be slightly larger than d_s. We may choose $R = \alpha d_s$, where $\alpha > 1$ is a small constant usually less than three. This way we ensure that not too many points are examined by the augmented DLD. This choice requires the sphere decoder to carry out a second round of search using new radius R. If lattice reduction is performed, we may simply set $R = \alpha d(\mathbf{t}, \hat{\mathbf{x}}) \geq \alpha d_s$, where $\hat{\mathbf{x}}$ is the output of LLL-ZF or LLL-SIC. Only a single round of search is needed by this approach. While this choice is weaker, the complexity is not much increased, as the LLL-based decision coincides with that of sphere decoding with high probability.

Our empirical results suggest that the overall complexity of augmented DLD is on the same order of the LLL algorithm. It is simpler than another improved decoder based on basis reduction presented in [21], whose complexity is a multiple of that of the LLL algorithm (it works for a finite lattice though).

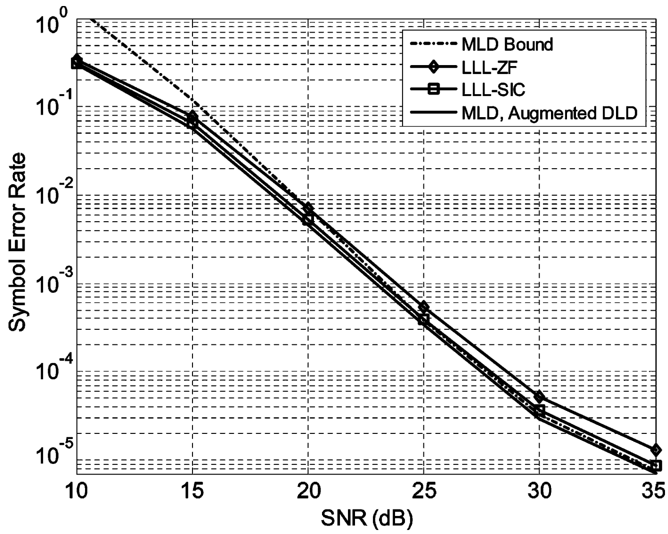

Fig. 3. Symbol error rate of conventional DD for $R = 2, n_T = 4, n_R = 1$, and $f_d = 0.0025$.

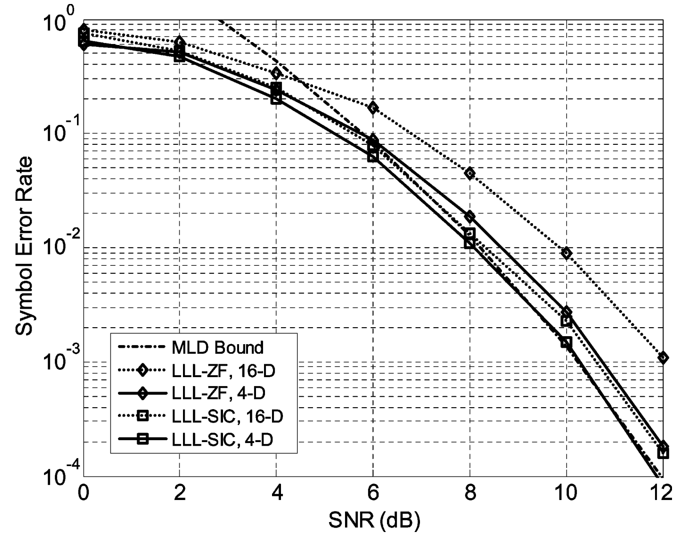

Fig. 4. Symbol error rate of conventional DD for $R = 2, n_T = n_R = 4$, and $f_d = 0.0025$ with LLL-ZF, LLL-SIC in 16-D and 4-D lattices.

IV. Numerical Results

In this section, we compare the performance and complexity of different schemes of conventional DD and DF-DD for diagonal constellations. The performance index is the error rate of DSTM symbols, as bit mapping is of secondary importance to this paper. The Jakes fading model is employed in computer simulation. Constellations are drawn from [11, Table I] if available. Clearly, the performance of sphere decoding is independent of what kind of preprocessing or ordering is used, though the complexity may vary significantly. The Schnorr–Euchner strategy is followed in augmented DLD, and we set the initial radius as $R = 2d(\mathbf{t}, \hat{\mathbf{x}})$, where $\hat{\mathbf{x}}$ is the output of LLL-SIC. Surprisingly, with this simple choice of R we have not observed a single discrepancy between MLD and augmented DLD, throughout all comparisons made in simulation. If MLD is deemed too time consuming, we plot the union bound for MLD derived in [15], which is sufficiently tight at high SNR. For DF-DD, the depicted union bound is of a genie-aided (i.e., correct feedback) detector, which is often a good approximation of the real-error rate.

Fig. 3 shows the performance of conventional DD for $R = 2$, $n_T = 4$, $n_R = 1$, and $f_d = 0.0025$. Our results slightly differ from [12, Fig. 5] at high SNR, but the simulated performance for MLD has excellent match with the analytic union bound. It is seen that LLL-SIC has better performance than LLL-ZF. Note also that the performance of LLL-SIC is close to that of MLD, which is typical of low-dimensional DD. The decisions of augmented DLD and MLD are compared verbatim; no difference is found. Thus, their performance is depicted by the same line.

Conventional DD utilizing an n_T-D lattice is compared with $n_T n_R$-D lattice decoding in Fig. 4, for the above setting but with $n_R = 4$. It is seen that LLL-ZF is susceptible to the lattice dimension; decoding in a 16-D lattice leads to performance loss of about 1.5 dB at high SNR, despite its higher complexity. In comparison, LLL-SIC differs by 0.5 dB only. Again, the performance of LLL-SIC in a four-dimensional (4-D) lattice is close to MLD. In addition, we can see that when the lattice dimension is reduced by a factor of 4, the performance gap between LLL-ZF and MLD is reduced by a factor of 4. This is no coincidence; it is in complete agreement with the analysis in the Appendix.

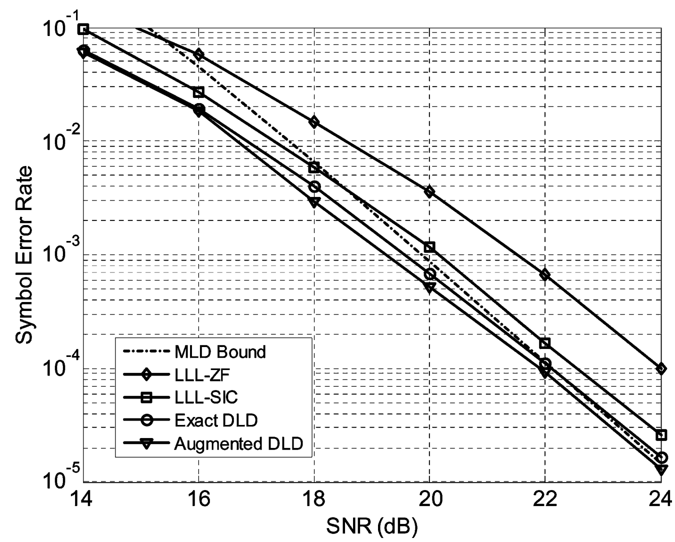

Fig. 5. Symbol error rate of conventional DD for $R = 2, n_T = 8, n_R = 1$, and $f_d = 0.001$.

The performance gain of exact and augmented DLD in conventional DD is apparent in Fig. 5 for $R = 2$, $n_T = 8$, $n_R = 1$, and $f_d = 0.001$. We just pick a good (but not necessarily the optimum) constellation $\mathbf{u} = [1\ 1551\ 3693\ 5951\ 10593\ 10643\ 25213\ 29893]^T$ found in random search. It is seen that augmented DLD is over 0.5 dB better than LLL-SIC at high SNR, and the gap between LLL-ZF and LLL-SIC is expanded to 1.5 dB. Exact DLD lies between LLL-SIC and augmented DLD. Remarkably, the performance of augmented DLD approaches the union bound for MLD as the SNR grows. It means that the performance loss due to the cosine approximation is practically recovered. Since this constellation size ($L = 65536$) is very large, we only compare 100 trials of MLD with augmented DLD for each SNR. Every trial sees exactly the same decisions.

TABLE I
PER-SYMBOL COMPLEXITY (IN KILOFLOPS) OF CONVENTIONAL DD OVER A CHANNEL WITH $f_d = 0.0025$ AND $\mathrm{SNR} = 20$ dB

n_T	R_c	L	MLD	DLD w/ LLL Reduction				Sphere Decoding w(/o) Ordering		
				LLL-ZF	LLL-SIC	Exact	Augmented	w/o	Norm	V-BLAST
2	1	4	0.09	0.21	0.30	0.28	0.40	0.14	0.17	0.30
3	1	8	0.24	0.60	0.84	0.81	1.07	0.33	0.43	0.83
4	1	16	0.60	1.34	1.87	1.74	2.42	0.68	0.86	1.73
5	1	32	1.46	2.36	3.41	3.14	4.17	1.24	1.53	3.14
6	1	64	3.48	3.91	5.71	5.26	6.96	2.38	2.56	5.16
2	2	16	0.31	0.26	0.34	0.32	0.47	0.14	0.17	0.30
3	2	64	1.75	0.75	1.04	0.97	1.30	0.42	0.46	0.85
4	2	256	9.24	1.62	2.23	2.12	2.67	1.29	1.07	1.83
5	2	1024	46.1	3.27	4.29	4.08	5.17	6.39	3.51	4.18
6	2	4096	221	5.13	7.08	6.29	8.31	18.1	9.36	8.31
2	3	64	1.17	0.28	0.36	0.33	0.49	0.16	0.20	0.32
3	3	512	13.8	0.68	0.93	0.85	1.37	0.58	0.62	1.06
4	3	4096	147	1.71	2.25	2.05	3.01	3.32	2.30	2.06
5	3	32768	1470	3.50	4.54	4.28	5.96	26.7	32.1	17.1
6	3	262144	13000	5.87	6.81	6.89	10.6	118	182	105

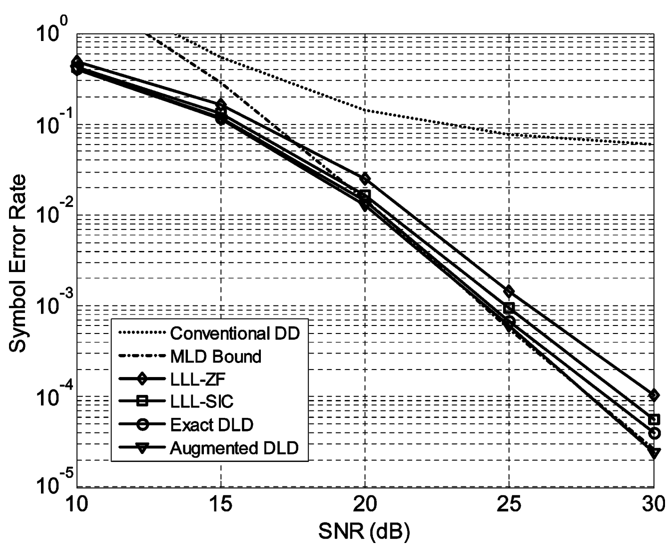

Fig. 6. Symbol error rate of DF-DD for $R = 2, n_T = 5, n_R = 1, f_d = 0.01$, and $N = 3$.

Fig. 6 illustrates the performance of DF-DD for $R = 2, n_T = 5$, $n_R = 1$, $f_d = 0.01$, and $N = 3$. The union bound for conventional DD is also included as a benchmark. Once again no different decoding outcomes of augmented DLD and MLD are observed. It is seen that LLL-SIC improves over LLL-ZF by 1 dB at high SNR. The augmented DLD improves further by 1 dB, and agrees with the MLD bound excellently.

Generally, the improvement of exact and augmented DLD tends to be more visible in high dimensions or DF-DD. The former is an effect of the respective approximation of the Voronoi cell; the latter might be an effect of the feedback of occasionally erroneous decisions.

To evaluate the computational complexity, we count the average number of flops required to decode each incoming DSTM symbol. This measure is quite meaningful, as it is largely independent of the particular processor and programming environment. Moreover, it reflects the real hardware complexity more objectively than other measures such as CPU running time. Table I shows the average numbers of flops per detected symbol for various schemes of conventional DD over a channel with $f_d = 0.0025$ and $\mathrm{SNR} = 20$ dB. The similar trend has been observed for DF-DD. The ratio between the data for MLD and LLL-ZF differs somewhat from [12], but the trend is the same. It is clear that LLL-ZF, LLL-SIC and exact DLD with LLL reduction have similar complexity. Since we do not store the orthogonal vectors in LLL reduction, the QR decomposition makes LLL-SIC slightly more complex than LLL-ZF. Augmented DLD with LLL reduction and SIC initial radius has less than the double complexity of LLL-ZF. All of these four algorithms are faster than MLD for constellation size $L \geq 64$. Ordering[3] is not effective in complexity reduction of sphere decoding here. This is because the advantage of ordering in complexity reduction will not appear until the dimension is quite large ($\geq$30) [3, Fig. 8]. Sphere decoding with or without ordering is faster than LLL reduction-aided DLD for small constellation sizes. For large constellations sizes, the LLL reduction makes DLD significantly faster, which is the case of more interest in high data rate MIMO communication. It can be seen that the complexity of LLL reduction-aided decoding increases much slower than $O(n_T^4)$ as predicted by theory, for practical numbers of transmit antennas. Another advantage of LLL reduction-aided decoding is that the complexity is less dependent on SNR.

V. CONCLUSION AND DISCUSSION

We have presented improved fast decoding algorithms for multiantenna differential modulation. We showed the two basis reduction-aided approximate DLD algorithms, i.e., LLL-ZF and LLL-SIC, exhibit different performance and gave a quantitative analysis. In particular, the latter is much better in high dimensions. Sphere decoding was applied to find the closest point

[3]The V-BLAST ordering is implemented, as in [19].

in the Euclidean space exactly. Furthermore, we proposed augmented DLD to compensate for the remaining cosine approximation, which practically achieves the MLD performance. The performance gain of exact and augmented DLD is clear for large values of n_T and high SNR. Moreover, our definition of an n_T-D lattice for the MIMO differential scheme is simple, yet effective in performance improvement and complexity reduction.

Various methods of preprocessing and ordering were examined, and we found that LLL reduction is most effective in reducing the overall complexity for large constellation sizes. The four LLL reduction-aided DLD algorithms have complexity on the same order, and are all much faster than brute-force MLD for large constellation sizes. The small amount of extra complexity of DLD is rewarding in terms of the obtained performance gain. Ordinarily, basis reduction is seen as the preprocessing stage, with minor complexity, for sphere decoding. Here, a different viewpoint is more suitable for multiantenna DLD: sphere decoding as a postprocessing stage of basis reduction. In contrast to coherent MIMO decoding in a quasi-static fading channel [2], [3], the LLL algorithm is often the decoding-speed bottleneck of multiantenna DLD. The application of some fast LLL-type reduction algorithms may warrant a serious investigation.

The empirical evidence of the identical outcomes of MLD and augmented DLD indicates that there may exist a lower bound on the value of α for the initial radius $R = \alpha d_s$, above which augmented DLD is strictly optimum. Such a lower bound would depend on the geometric relation of $\mathcal{V}^{\mathrm{ML}}$ and $\mathcal{V}^{\mathrm{eucl}}$, and the covering radius of the lattice.

While we only dealt with diagonal constellations in this paper, DLD allows for a straightforward extension to many nondiagonal constellations. This is because most nondiagonal group codes contain large diagonal subgroups. We may apply DLD within each subgroup and use an exhaustive method across subgroups (cf. [13], [14]). Finally, sphere decoding has recently been applied to single-antenna multiple-symbol DD (MSDD) [22]; a similar extension of DLD to MSDD for DSTM would be of great interest.

Appendix
Upper Bounds on Performance Loss of LLL-ZF/SIC

In this appendix, we show the power-efficiency loss of LLL-ZF/SIC is upper-bounded by a constant that is a function of the dimension n of a lattice only. To do this, we derive lower bounds on the MSEDs of the decoders for a given lattice, which dominates the performance at high SNR.

Confer the decision regions in Fig. 1. The MSED of MLD is obviously $d^2_{\mathrm{ML}} = \lambda^2/4$, where λ denotes the length of the shortest nonzero vector in the lattice with basis $\mathbf{B}$. Let Θ_i be the angle between $\mathbf{b}_i$ and the linear space spanned by the other $n-1$ basis vectors, and $\hat{\mathbf{B}} = [\hat{\mathbf{b}}_1, \ldots, \hat{\mathbf{b}}_n]$ be the Gram-Schmidt matrix of $\mathbf{B}$. The MSEDs of LLL-ZF and LLL-SIC are, respectively, given by

$$\begin{aligned} d^2_{\mathrm{ZF}} &= \frac{1}{4} \min_{1\le i\le n} |\mathbf{b}_i|^2 \sin^2 \Theta_i \\ d^2_{\mathrm{SIC}} &= \frac{1}{4} \min_{1\le i\le n} |\hat{\mathbf{b}}_i|^2. \end{aligned} \tag{8}$$

Babai proved $\sin\Theta_i \ge (\sqrt{2}/3)^n$ for an LLL-reduced lattice [17, eq. (5.1)], which in this context means

$$d^2_{\mathrm{ZF}} \ge \left(\frac{2}{9}\right)^n \cdot \frac{1}{4} \min_{1\le i\le n} |\mathbf{b}_i|^2 \ge \left(\frac{2}{9}\right)^n \cdot \frac{\lambda^2}{4} = \left(\frac{2}{9}\right)^n d^2_{\mathrm{ML}}. \tag{9}$$

Moreover, the LLL-reduced lattice satisfies $|\hat{\mathbf{b}}_i|^2 \ge 2^{-(i-1)} |\mathbf{b}_1|^2$ [6, eq. (1.7)]. Hence, we have

$$d^2_{\mathrm{ZF}} \ge 2^{-(n-1)} \cdot \frac{1}{4} |\mathbf{b}_1|^2 \ge 2^{-(n-1)} \cdot \frac{\lambda^2}{4} = 2^{-(n-1)} d^2_{\mathrm{ML}}. \tag{10}$$

The above analysis implies that the losses in power efficiency are upper-bounded by 4.5^n and $2^{(n-1)}$, respectively, in a linear Gaussian channel. Even if (5) does not exactly conform to the linear Gaussian model, we suspect its performance can be characterized in a similar way. This analysis for a fixed but arbitrary basis $\mathbf{B}$ is obviously extendible to an ensemble of random fading coefficients. Therefore, we conclude the performance losses are less than $6.5n$ dB for LLL-ZF and $3n$ dB for LLL-SIC, respectively.

It is worth mentioning that the analysis is generally applicable to lattice decoding problems in linear Gaussian channels. For example, it explains why lattice-reduction-aided decoding achieves full diversity of a spatial multiplexing system, as often observed in computer simulation [2], [21].

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