AC network state estimation using linear measurement functions

R.A. Jabr and B.C. Pal

Abstract: The real/reactive power and current magnitude measurements can be accounted for in an AC network state estimator using linear measurement functions. The nonlinearity in the conventional AC state estimator equations is transferred from the measurement functions into a system of nonlinear equality constraints which is independent of the measurement set. The new format of equations entails two advantages. First, it can be easily integrated in optimisation routines which employ first- and second-order derivative functions. Second, the linear measurement functions can benefit from scaling techniques which are well documented in the linear programming literature. This research details the implementation of a least absolute value state estimator employing the new format of equations. The optimisation method is based on a primal-dual interior-point method that can accurately account for zero injection measurements and power directions. Numerical testing is used to validate the approach for networks with measurement sets that are (i) conventional and (ii) have a high proportion of current magnitude measurements and power signs.

List of symbols

$b_{sh}/2$	1/2 charging susceptance in the π equivalent model
b_{in}	series susceptance in the π equivalent model
B_{in}	imaginary part of $\hat{Y}_{in}$
g_{in}	series conductance in the π equivalent model
G_{in}	real part of $\hat{Y}_{in}$
h_k^{T}	row vector of the kth linear measurement function
I_{in}	magnitude of the line (i–n) current leaving bus i
m	number of measurements in the network
N	number of buses in the network
P_i	real power injection at bus i
P_{in}	real power line (i–n) flow leaving bus i
Q_i	reactive power injection at bus i
Q_{in}	reactive power line (i–n) flow leaving bus i
r_k, s_k	positively bounded variables in the least absolute value representation
RMS-VE	root mean square voltage error
RMS-AE	root mean square angle error
REL-VE	maximum relative voltage error
REL-AE	maximum relative angle error
R_{in}	variable in the proposed power flow format defined as $V_iV_n\cos(\delta_i - \delta_n)$
T_{in}	variable in the proposed power flow format defined as $V_iV_n\sin(\delta_i - \delta_n)$
u_i	variable in the proposed power flow format defined as $V_i^2/\sqrt{2}$
V_{i}	voltage magnitude at bus i
$\boldsymbol{x}$	state vector
$\boldsymbol{x}_i^{\mathrm{e}}$	estimated value of $\boldsymbol{x}_i$
$\boldsymbol{x}_i^{\mathrm{tr}}$	true value of $\boldsymbol{x}_i$
$\hat{Y}_{in}$	complex representation of an element in the bus admittance matrix
z_k	kth measurement
$z_k^{\min}$, $z_k^{\max}$	minimum and maximum values of the kth measurement
δ_i	voltage angle at bus i

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1 Introduction

Governments of many industrialised nations have pledged to reduce their carbon emissions in accordance with the Kyoto agreement on climate change [1]. For instance, UK government has set a carbon cut target of 60% by 2050 [2]. Reducing carbon emissions requires governments and power utilities to invest in renewable energy sources such as tidal, solar and wind power. These sources are by nature geographically dispersed and comparatively small sized which in turn necessitates their connection to distribution networks. Consequently, there is an emerging need for integrated monitoring and control of both transmission and distribution networks.

State estimation is the main function for monitoring power networks. Since the 1970s, it has been researched in the context of transmission networks [3, 4]. In transmission state estimation, conventional measurement sets are composed of real/reactive power pairs and voltage magnitudes. Current magnitude measurements are occasionally employed for increasing the level of redundancy. Distribution networks, on the other hand, have limited conventional measurements. In many cases, the measurement sets for such networks are dominated by current magnitude measurements. These measurements are known to complicate the state estimation problem because they do not contain directional information. The lack of power flow directions implies that although a network may be numerically observable in the conventional sense (measurement Jacobian is of full column rank), its state estimation solution may not be unique [5]. The literature reports examples of networks having a large proportion of current magnitude measurements but that can be still made uniquely observable through incorporating into the constraint set power injection signs and zero injection pseudo-measurements [6]. Therefore for AC network state estimation employing current magnitude measurements, the computational engine should be capable of handling functional equality/inequality constraints.

The underlying computational procedure in any state estimator is an optimisation function. In the state estimation literature, the employed optimisation functions can be classified as either first- or second-order methods, depending on the order of the derivative. The first-order methods include the classical weighted least squares [7], the iteratively re-weighted least squares [8–11] and the linear programming based least absolute value estimator [12]. The second-order methods require the evaluation of the Lagrangian Hessian matrix. The most recent second-order state estimation implementations rely on primal-dual interior-point methods. They have been implemented in the context of a least absolute value estimator [13] and a Huber M-estimator [14]. Moreover, the interior-point implementations have been shown to be successful in enforcing zero injection measurements and load power limits [14]. Reference [15] reports a least absolute value interior-point implementation which also accounts for current magnitude measurements. In practice, the upgrade of an interior-point based state estimation function to account for current magnitude measurements on top of the conventional measurement set requires significant time and effort for coding and subsequently testing the computation of the Jacobian and Lagrangian Hessian matrices.

This paper proposes a primal–dual interior-point implementation of a least absolute value AC network state estimator that uses linear measurement functions. These functions have a constant Jacobian matrix and therefore do not contribute to the Lagrangian Hessian. In the proposed format, the nonlinearity of the state-estimation function is embodied in a fixed set of equality constraints. This set of nonlinear constraints is the only one which contributes to the Langrangian Hessian computation. The paper shows that the new format of equations can be obtained using simple variable substitutions. A similar format of equations was first proposed for radial networks in [16] and then shown to be equivalent to a second-order cone program in [17].

2 Power flow equations

Let $\hat{Y}_{in} = G_{in} + jB_{in}$ denote the complex rectangular representation of an element in a $N \times N$ bus admittance matrix. If bus voltages are expressed in polar form ($\tilde{V}_i = V_i \angle \delta_i$), the real and reactive injected powers at an arbitrary bus i [18] are

$$P_i = V_i^2 G_{ii} + \sum_{\substack{n=1 \\ n \neq i}}^{N} \Big[V_i V_n G_{in} \cos(\delta_i - \delta_n) + V_i V_n B_{in} \sin(\delta_i - \delta_n) \Big] \quad (1)$$

$$Q_i = -V_i^2 B_{ii} + \sum_{\substack{n=1 \\ n \neq i}}^{N} \Big[V_i V_n B_{in} \cos(\delta_i - \delta_n) - V_i V_n G_{in} \sin(\delta_i - \delta_n) \Big] \quad (2)$$

By following [17], define $R_{in} = V_i V_n \cos(\delta_i - \delta_n)$, $T_{in} = V_i V_n \sin(\delta_i - \delta_n)$ and $u_i = V_i^2/\sqrt{2}$. The nonlinear power flow equations become

$$P_i = \sqrt{2} G_{ii} u_i + \sum_{\substack{n=1 \\ n \neq i}}^{N} [G_{in} R_{in} + B_{in} T_{in}] \quad (3)$$

$$Q_i = -\sqrt{2} B_{ii} u_i - \sum_{\substack{n=1 \\ n \neq i}}^{N} [B_{in} R_{in} - G_{in} T_{in}] \quad (4)$$

From the above definitions of R_{in}, T_{in} and u_i, it follows that

$$2u_i u_n = R_{in}^2 + T_{in}^2 \quad (5)$$

$$\delta_i - \delta_n = \tan^{-1}\left(\frac{T_{in}}{R_{in}} \right) \quad (6)$$

3 State estimator

Let $\boldsymbol{x}$ denote the state vector

$$\boldsymbol{x} = [\ldots, u_i, \ldots, R_{ij}, \ldots, T_{ij}, \ldots, \delta_i, \ldots]^{\mathrm{T}} \quad (7)$$

where it is understood that $u_i \geq 0$ and $R_{ij} \geq 0$.

The least absolute value state estimation problem can be expressed as

$$\text{minimise} \sum_{k=1}^{m} \left| h_k^{\mathrm{T}} \boldsymbol{x} - z_k \right| \text{subject to} \quad (8)$$

$$\text{equality constraints: } h_k^{\mathrm{T}} \boldsymbol{x} = z_k, \quad (9)$$

$$\text{inequality constraints: } z_k^{\min} \leq h_k^{\mathrm{T}} \boldsymbol{x} \leq z_k^{\max}, \quad (10)$$

feasibility constraints: equations (5) and (6).

In the above problem, h_k^{T} is the kth row of the linear measurement function matrix, z_k represents the kth measurement ($k = 1, \ldots, m$), (9) models the zero-injection pseudo-measurements and (10) constrains the direction of the power injection or flow. The inequality constraints (10) are commonly used in conjunction with the current magnitude measurements. The measurement functions take one of the following forms, depending on the measurement type.

1. Real power injection measurement P_i

$$h_k^{\mathrm{T}} \boldsymbol{x} = \sqrt{2} G_{ii} u_i + \sum_{\substack{n=1 \\ n \neq i}}^{N} [G_{in} R_{in} + B_{in} T_{in}] \quad (11)$$

2. Reactive power injection measurement Q_i

$$h_k^{\mathrm{T}}\boldsymbol{x} = -\sqrt{2}B_{ii}u_i - \sum_{\substack{n=1\\ n\neq i}}^{N}[B_{in}R_{in} - G_{in}T_{in}] \quad (12)$$

3. Real power flow measurement P_{in}

$$h_k^{\mathrm{T}}\boldsymbol{x} = \sqrt{2}g_{in}u_i - g_{in}R_{in} - b_{in}T_{in} \quad (13)$$

where g_{in}, b_{in}: series conductance and susceptance in the π equivalent model.

4. Reactive power flow measurement Q_{in}

$$h_k^{\mathrm{T}}\boldsymbol{x} = -\sqrt{2}\left(b_{in} + \frac{b_{sh}}{2}\right)u_i + b_{in}R_{in} - g_{in}T_{in} \quad (14)$$

where $b_{sh}/2$: $1/2$ charging susceptance in the π equivalent model.

5. Current magnitude measurement I_{in} transformed into I_{in}^2 [6]

$$h_k^{\mathrm{T}}\boldsymbol{x} = \sqrt{2}Au_i + \sqrt{2}Bu_n - 2CR_{in} + 2DT_{in} \quad (15)$$

where

$$A = g_{in}^2 + \left(b_{in} + \frac{b_{sh}}{2}\right)^2,$$

$$B = g_{in}^2 + b_{in}^2,$$

$$C = g_{in}^2 + b_{in}\left(b_{in} + \frac{b_{sh}}{2}\right),$$

$$D = \frac{g_{in}b_{sh}}{2}$$

6. Voltage magnitude measurement V_i transformed into $V_i^2/\sqrt{2}$

$$h_i^{\mathrm{T}}\boldsymbol{x} = u_i \quad (16)$$

The above linear measurement functions can account for additional state variables from load-flow control devices such as tap-changing or phase-shifting transformers provided that these devices are represented using the power injection model [19].

4 Interior-point solver

It is well known that there are two mathematically equivalent nonlinear programming representations of the direct least absolute value state estimation problem (5), (6), (8)–(10). Both representations have their objective and constraint functions twice continuously differentiable and are therefore computationally tractable using interior-point methods [13]. The first representation replaces (8) by a linear objective function and functional inequality constraints [13]

$$\text{minimise} \sum_{k=1}^{m} s_k \text{ subject to} \quad (17)$$

$$-s_k \leq h_k^{\mathrm{T}}\boldsymbol{x} - z_k \leq s_k; \quad k = 1, \ldots, m \quad (18)$$

The second representation substitutes (8) with a linear objective function, functional equality constraints and positively bounded variables [13]

$$\text{minimise} \sum_{k=1}^{m} (r_k + s_k) \text{ subject to} \quad (19)$$

$$h_k^{\mathrm{T}}x - z_k + r_k - s_k = 0, \quad (20)$$

$$r_k \geq 0,\ s_k \geq 0;\ k = 1, \ldots, m \quad (21)$$

Numerical experiments reported in [13] have shown that for least absolute value state estimation, the second formulation results in a more numerically robust interior-point implementation.

In this study, the direct state estimation problem is represented using (19–21) and solved via the primal–dual interior-point method described in [13]. The interior-point solver requires, at each iteration, the computation of the Jacobian and Lagrangian Hessian matrices. The Jacobian matrix corresponding to the measurements (19) together with the equality (9) and inequality constraints (10) is constant throughout all iterations. For the feasibility constraints (5), (6), the formulae for computing the Jacobian and Hessian elements are given in the appendix. The initial starting vector is chosen as follows

$r_k = s_k = 1$ for all measurements
$u_i = 1/\sqrt{2}$ and $\delta_i = 0$ for all nodes
$R_{ij} = 1$ and $T_{ij} = 0$ for all lines

The solver terminates when all the stopping criteria are satisfied with a tolerance of 10^{-8}.

4.1 Scaling

In linear programming, scaling with positive factors is a general conditioning transformation that can be applied to the matrix of constraints without significantly changing the problem definition. The most widely used method is scaling rows and columns to have unit norms [20]. The use of scaling in linear programming based least absolute value state estimation was first investigated in [21]. According to the numerical results in [21], normalisation is the most effective provided that (i) column scaling is performed before row scaling and (ii) the unitary columns corresponding to the positively bounded variables (r_k and s_k in (20)) are preserved. Advantages of scaling include a reduction in the number of linear programming iterations and a decrease in the number of leverage points [21].

The numerical experiments conducted in this study have shown that scaling the measurement matrix such that its rows have unit length results in improved convergence and a reduced number of interior-point iterations. In fact, some problems failed to converge without scaling. As in [21], the normalisation did not include the unitary columns corresponding to the variables r_k and s_k in (20). In any case, column scaling was not implemented because the variables in the linear measurement equations also appear in the nonlinear feasibility constraints.

5 Numerical results

A prototype implementation of the proposed estimator was programmed in MATLAB running on a Pentium IV, 1.89 GHz PC with 256 Mbytes of RAM. Testing was carried out on the IEEE 14, 30 and 118 bus systems. The line and load data for these systems are available from [22]. For each system, the true values of the measurements were generated using a Newton–Raphson load flow algorithm. The noise was added to the true values assuming

Table 1: Measurement sets adapted from [23]

Measurements and constraints	IEEE 14	IEEE 30
real injection	2, 4, 5, 6, 10, 11, 12, 13, 14	1, 2, 3, 5, 8, 10, 14, 15, 17, 20, 21, 23, 24, 26
reactive injection	2, 4, 5, 6, 8, 10, 11, 12, 13, 14	1, 2, 3, 5, 8, 10, 13, 14, 15, 17, 20, 21, 23, 24, 26
real zero injection	7, 8	6, 9, 11, 13, 22, 25, 27, 28
reactive zero injection	7	6, 9, 22, 25, 27, 28
real/reactive power flow	1–2, 2–3, 4–7, 5–6, 6–11, 6–13, 7–8, 7–9, 9–14	1–2, 1–3, 2–5, 2–6, 6–10, 9–11, 4–12, 16–17, 18–19, 19–20, 24–25, 27–30, 29–30, 6–28, 13–12, 26–25.
voltage magnitude	2, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14	1, 2, 3, 5, 6, 8, 9, 10, 13, 14, 15, 17, 20, 21, 23, 24, 26, 27, 28

Gaussian distribution with zero mean and a standard deviation of 0.01 pu for power/current measurements and 0.005 pu for voltage measurements. The state estimator was tested under two measurement scenarios: first for conventional measurement sets and second for measurement sets with current magnitudes and power signs.

5.1 Conventional measurement sets

The conventional measurement sets for the IEEE 14 and 30 bus systems were adapted from [23]. The observability algorithm in [23] relies on the DC network model. The corresponding measurement sets for the AC network model are given in Table 1. For the IEEE 118 bus test system, a simple measurement set was used. It consisted of (i) the voltage magnitude at the slack node and (ii) the real and reactive power flows at one end of each line, that is estimation was carried out using a line-only state estimator.

Table 2 shows the following performance metrics for each of the above systems simulated in the presence of measurement noise

$$\text{RMS voltage error (pu): RMS-VE} = \sqrt{\frac{\sum_{i=1}^{N}(V_i^{\text{tr}} - V_i^{\text{e}})^2}{N}}$$

$$\text{RMS angle error (rad): RMS-AE} = \sqrt{\frac{\sum_{i=1}^{N}(\delta_i^{\text{tr}} - \delta_i^{\text{e}})^2}{N}}$$

Maximum relative voltage error (%):

$$\text{REL-VE} = \max_{i=1,\ldots,N}\left|\frac{V_i^{\text{tr}} - V_i^{\text{e}}}{V_i^{\text{tr}}}\right| \times 100$$

Table 2: State estimator performance – conventional measurement set with noise

Network	RMS-VE	RMS-AE	REL-VE	REL-AE	TIME (s)	ITER.
IEEE14	2.77×10^{-3}	2.67×10^{-3}	0.43	0.85	0.21	9
IEEE30	4.25×10^{-3}	4.39×10^{-3}	0.88	1.39	0.26	10
IEEE118	2.62×10^{-3}	1.91×10^{-3}	0.61	0.72	0.62	12

Maximum relative angle error (%):

$$\text{REL-AE} = \max_{i=1,\ldots,N}\left|\frac{\delta_i^{\text{tr}} - \delta_i^{\text{e}}}{\delta_i^{\text{tr}}}\right| \times 100$$

CPU time (s): TIME
Number of interior-point iterations: ITER.

In the above equations, the superscript tr designates the true value and e represents the estimated value. Note that in order to avoid division by zero in the REL-AE equation, the angle at the slack node was set to 1 rad. The value of 1 rad was chosen because it allows meaningful comparison between REL-VE and REL-AE in Table 2. In the absence of measurement noise, the state estimator converged to the true solution. In fact, all values of REL-VE and REL-AE in Table 2 were at 0.00%.

Table 3 shows similar results for the test systems in the presence of bad data. It is important to note that the error values in Tables 2 and 3 are in agreement with the range of errors from a previous study reporting the

Table 3: State estimator performance – conventional measurement set with noise and bad data

Network	RMS-VE	RMS-AE	REL-VE	REL-AE	TIME (s)	ITER.
IEEE14	4.93×10^{-3}	3.74×10^{-3}	1.39	1.17	0.25	11
IEEE30	5.07×10^{-3}	9.00×10^{-3}	1.43	3.50	0.26	9
IEEE118	5.95×10^{-3}	3.25×10^{-3}	1.04	1.21	0.59	12

Table 4: True, measured and estimated values

Network	Quantity	True value	Measured value	Estimated value
IEEE14	P_{5-6}	0.438998	0.938998	0.462994
	Q_{5-6}	−0.123030	−0.623030	−0.114999
IEEE30	P_1	2.609207	3.109207	2.608240
	Q_1	−0.203061	0.296939	−0.284769
IEEE118	P_{26-25}	0.901500	1.401500	0.926213
	Q_{26-25}	−0.915405	−0.415405	−0.864993

Table 5: Measurement sets adapted from [24] and [14]

Measurements and constraints	IEEE14	IEEE30
real injection	1, 2, 3, 4, 6, 9, 10, 12, 13	1, 2, 3, 5, 14, 16, 17, 26, 29, 30
reactive injection	1, 2, 3, 4, 6, 9, 10, 12, 13	1, 2, 3, 5, 11, 14, 16, 17, 26, 29, 30
real zero injection	7, 8	6, 9, 11, 13, 22, 25, 27, 28
reactive zero injection	7	6, 9, 22, 25, 27, 28
real/reactive power flow	1–2, 4–7, 4–9, 7–8, 7–9	1–2, 1–3, 2–4, 2–5, 2–6, 4–6, 6–7, 6–8, 6–9, 6–10, 4–12, 12–14, 12–15, 12–16, 19–20, 10–21, 10–22, 15–23, 24–25, 27–29, 27–30, 6–28, 2–1, 4–3, 6–2, 19–18, 24–22, 24–23, 27–28, 28–8, 28–6
voltage magnitude	1	1
current magnitude	6–11, 10–11	12–13, 10–20
real power injection sign	10	18, 20
reactive power injection sign	10	13, 18, 20

implementation of a least squares, a linear programming based least absolute value, and an iteratively re-weighted least squares estimator [11]. All the estimators in [11] use the standard power flow equations format. The bad data measurements together with their true and estimated values are given in Table 4. These results suggest that the proposed state estimator, like the conventional least absolute value estimator, is capable of rejecting bad data as long as they do not correspond to leverage point measurements.

As for the computational performance, comparisons with previous studies indicate that the execution time of the proposed estimator is less than the corresponding average execution times of the linear programming based least absolute value and the iteratively re-weighted least squares estimators (c.f. Tables 2 and 3 in [11]). Moreover, the execution time is comparable with that of the interior-point least absolute value state estimator employing the conventional nonlinear measurement functions (c.f. Tables 4 and 5 in [13]). Both the estimator in this paper and the one in [13] employ the same interior-point implementation.

5.2 Unconventional measurement sets

Unconventional measurement sets include current magnitudes and power signs. These can be useful for extending system observability [6]. For instance, [24] includes examples of measurement sets for the IEEE 14 and 30 bus test systems which do not provide complete network observability. To recover the overall system observability, [24] provides candidate locations for real/reactive power pseudo-measurements. The use of power pseudo-measurement intervals in the measurement sets of [24] was investigated earlier in [14]. In this research, the branches used in [14] for power pseudo-measurement placement were provided with current magnitude measurements and the power injection pseudo-measurements were replaced with inequality constraints indicating their signs. The corresponding measurement sets for the IEEE 14 and 30 bus test systems are given in Table 5. Table 6 shows the estimation results in the absence of noise. Table 7 shows similar results in the presence of measurement noise. It can be inferred from the results that the current magnitude measurements and power signs are useful for state estimation in this case.

When the current magnitude measurements constitute a very large proportion of the measurement set, state estimation is possible only if network observability can be guaranteed by other means. In certain examples, observability can be achieved by the use of zero injection pseudo-measurements and power signs [6]. The measurement configurations in [6] were also used in testing the state estimator. For the IEEE 14 bus system, the measurement set consisted of (i) voltage magnitudes at all nodes, (ii) line current magnitudes at both ends of every line and (iii) zero injection real/reactive pseudo-measurements. In addition, inequality constraints were used to enforce (i) the real power injection signs at all nodes except at node 2 were both load and generation are present and (ii) the reactive power injection signs for load nodes. For the IEEE 30 bus system, the measurement set similarly consisted of all voltage and line current magnitude measurements together with the zero injection pseudo-measurements. Again, real power injection signs were specified for all nodes and reactive power signs were defined for all load nodes. The estimation statistics in the presence of noise are shown in Table 8. It is evident that the current magnitude measurements lead to an increased number of interior-point iterations. This behaviour has

Table 6: State estimator performance – unconventional measurement set without noise

Network	RMS-VE	RMS-AE	REL-VE	REL-AE	TIME (s)	ITER.
IEEE14	8.31×10^{-7}	1.53×10^{-6}	0.00	0.00	0.34	25
IEEE30	7.15×10^{-5}	7.30×10^{-5}	0.02	0.03	0.28	10

Table 7: State estimator performance – unconventional measurement set with noise

Network	RMS-VE	RMS-AE	REL-VE	REL-AE	TIME (s)	ITER.
IEEE14	5.89×10^{-3}	2.03×10^{-3}	0.97	0.43	0.36	27
IEEE30	3.46×10^{-3}	1.50×10^{-3}	0.71	0.56	0.29	12

Table 8: State estimator performance –measurement set according to [6] with noise

Network	RMS-VE	RMS-AE	REL-VE	REL-AE	TIME (s)	ITER.
IEEE14	6.26×10^{-3}	8.69×10^{-3}	1.21	2.11	0.44	32
IEEE30	4.02×10^{-3}	7.08×10^{-3}	0.92	2.75	0.74	37

been previously reported in [15]. In AC networks with such type of measurement sets, the estimated state may be used for load estimation by computing the power delivered into each load node [25].

6 Conclusion

This paper presented a least absolute value state estimator based on a new power flow equations format. In this format, the real/reactive power and current magnitude measurements are modelled via linear functions. The nonlinearity is accounted for by a fixed set of nonlinear feasibility constraints. The proposed format can be easily integrated in optimisation functions that require second-order derivatives, for instance, a primal-dual interior-point solver. Numerical results show that with conventional measurement sets, the magnitude of error in the estimates is comparable to that from three standard state estimators which are based on the least squares, the linear programming least absolute value, and the iteratively re-weighted least squares techniques [11]. Even with unconventional measurement sets dominated by current magnitude measurements and power signs, the new estimator was shown to be capable of maintaining an acceptable level of error accuracy.

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8 Appendix

To compute the Jacobian and Lagrangian Hessian matrices corresponding to (5), it is first rewritten as

$$f = R_{in}^2 + T_{in}^2 - 2u_i u_n$$

The Jacobian and Hessian elements are

$$\frac{\partial f}{\partial R_{in}} = 2R_{in}, \ \frac{\partial f}{\partial T_{in}} = 2T_{in}, \ \frac{\partial f}{\partial u_i} = -2u_n, \ \frac{\partial f}{\partial u_n} = -2u_i,$$

$$\frac{\partial^2 f}{\partial R_{in}^2} = \frac{\partial^2 f}{\partial T_{in}^2} = 2, \ \frac{\partial^2 f}{\partial u_n \partial u_i} = \frac{\partial^2 f}{\partial u_i \partial u_n} = -2$$

Similarly, (6) is rewritten as

$$g = \delta_i - \delta_n - \tan^{-1}\frac{T_{in}}{R_{in}}$$

The corresponding Jacobian and Hessian elements are

$$\frac{\partial g}{\partial \delta_i} = 1, \ \frac{\partial g}{\partial \delta_n} = -1, \ \frac{\partial g}{\partial R_{in}} = \frac{T_{in}}{R_{in}^2 + T_{in}^2}, \ \frac{\partial g}{\partial T_{in}} = \frac{-R_{in}}{R_{in}^2 + T_{in}^2}, \ \frac{\partial^2 g}{\partial R_{in}^2} = \frac{-2R_{in}T_{in}}{(R_{in}^2 + T_{in}^2)^2}, \ \frac{\partial^2 g}{\partial T_{in}^2} = \frac{2R_{in}T_{in}}{(R_{in}^2 + T_{in}^2)^2}, \ \frac{\partial^2 g}{\partial R_{in}\partial T_{in}} = \frac{\partial^2 g}{\partial T_{in}\partial R_{in}} = \frac{R_{in}^2 - T_{in}^2}{(R_{in}^2 + T_{in}^2)^2}$$